RIT

Binding of Sparse Distributed Representations in Hierarchical Temporal Memory

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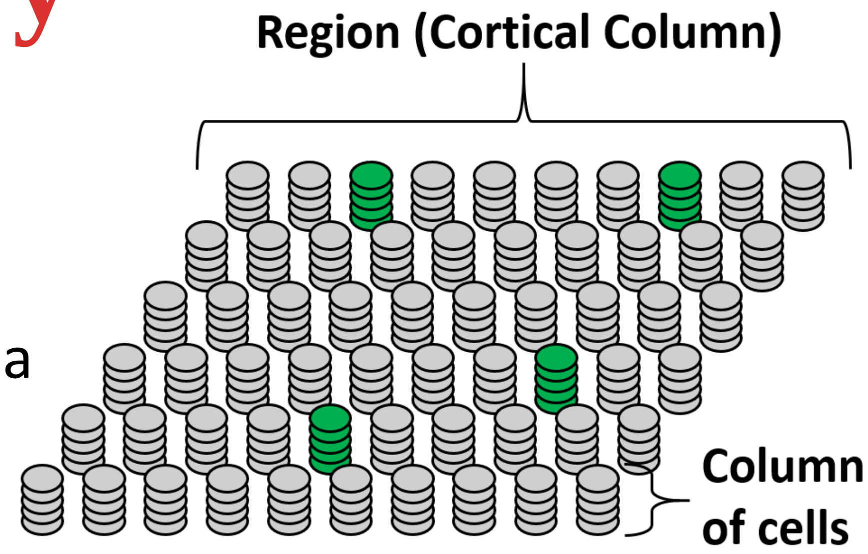
Hierarchical Temporal Memory

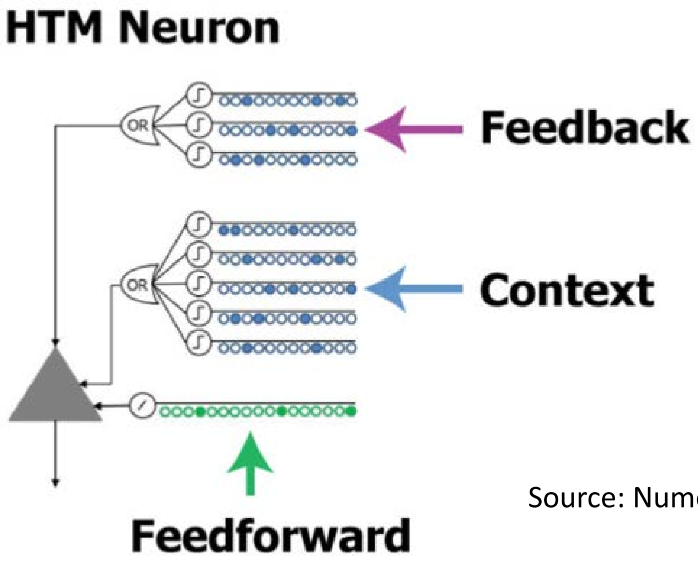


HTM Structure

- Pyramidal neurons (cells) form *Columns*
- A collection of *Columns* form a *Region*, which models a *Cortical Column* or layers 2/3 of the Neocortex

HTM theory is continually evolving

- HTM has two major algorithm portions
- Spatial Pooler - processes Feedforward Input
- Temporal Memory - processes Feedforward & Contextual Input
- Numenta *(Jeff Hawkins et. al)* is driving the HTM research

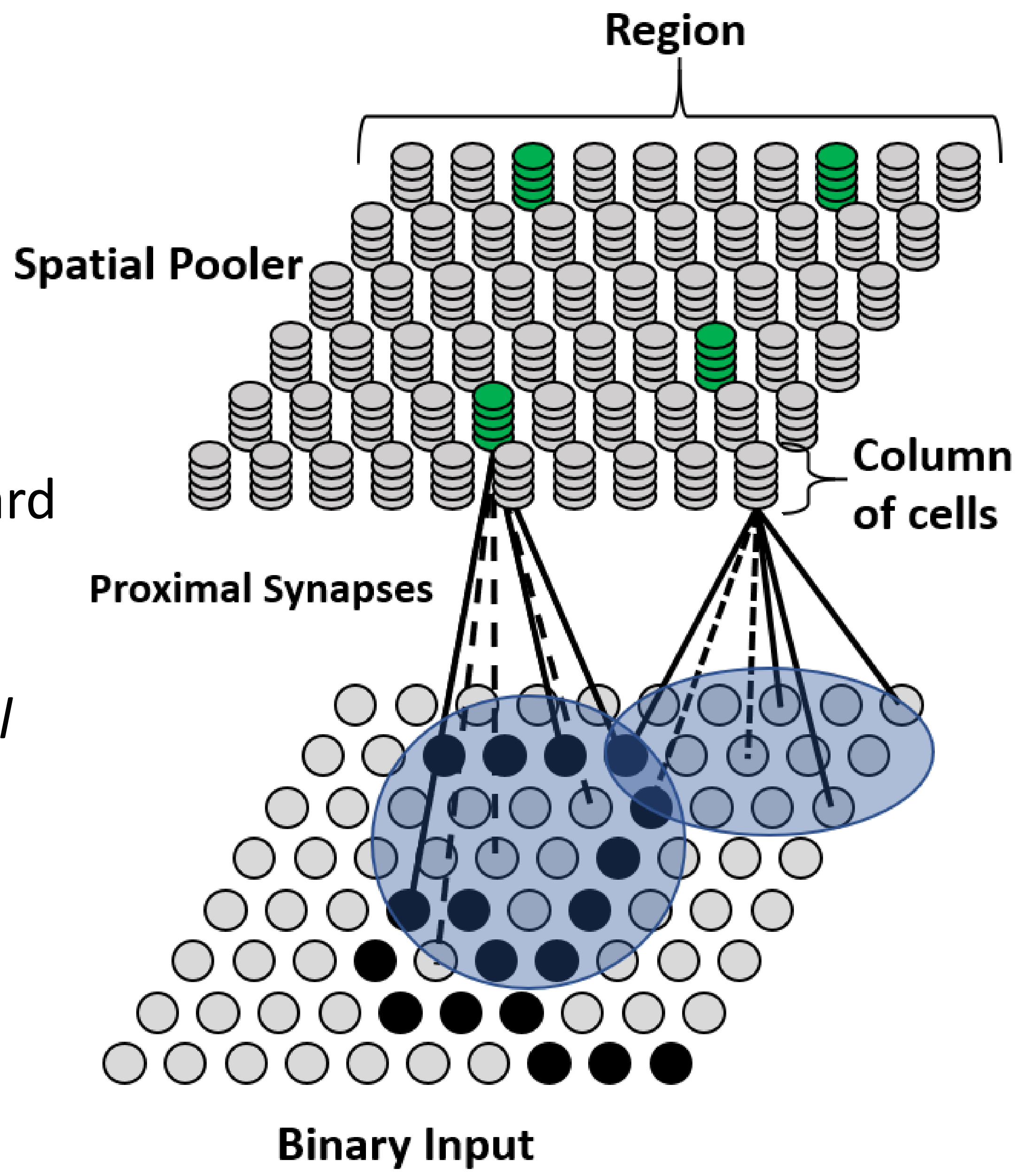


Source: Numenta

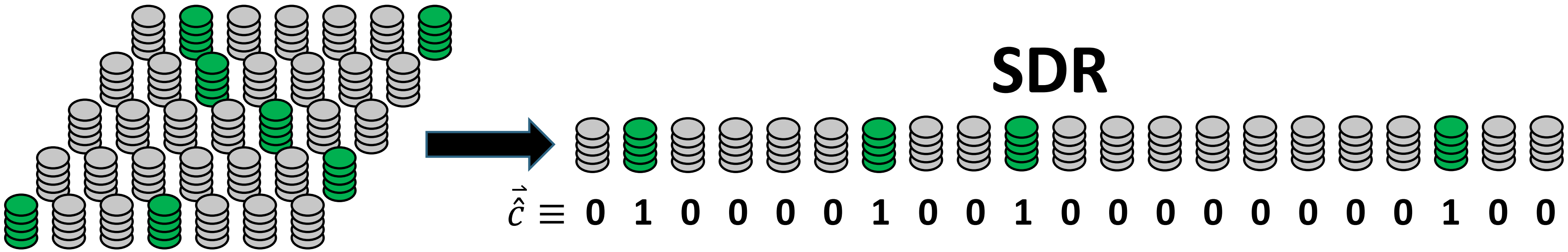
HTM Spatial Pooler

Spatial Pooler Structure

- *Columns* of *cells* become active with sufficient feedforward input
- Each *cell* in a *column* receives the same feedforward input
- Input flows through *proximal synapses*
- Hebbian learning governs connectivity of *proximal synapses*
- Online learning - adapts to changes in input data



Sparse Distributed Representation (SDR)



Sparse Distributed Representations are core part of HTM

- Large Binary Vector (2048 bits; the number of *columns* in the *Region*)
- Sparsity ~ 2% (low # of '1's)
- Distributed: non-localist and resilient to noise

Spatial Pooler learns to map similar inputs to similar outputs

- Two inputs that are similar should have some degree of overlap in their SDRs
- Similarity between SDRs can be computed with a dot product

Spatial Pooler Algorithm

Spatial Pooler Algorithm

1. Overlap
2. Inhibition
3. Learning

Notation:

- Input

$$\boldsymbol{X} \in \{0,1\}^{m \times q}$$

- Proximal Synapses

$$\boldsymbol{\Phi} \in \mathbb{R}^{m \times q}$$

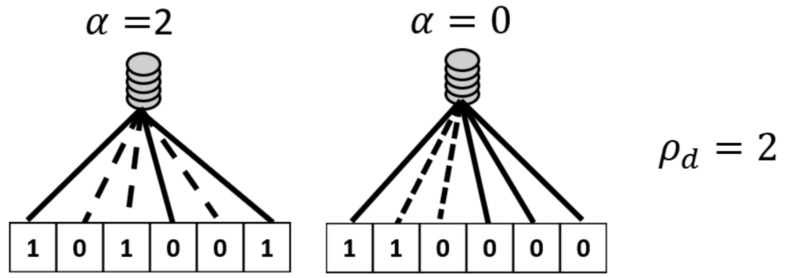
- Connected Synapses

$$\boldsymbol{Y} \equiv \mathrm{I}(\boldsymbol{\Phi}_i \geq \rho_s)\ \forall i$$

Overlap

$$\vec{\boldsymbol{\alpha}} \equiv \begin{cases} \vec{\hat{\alpha}}_i \vec{\boldsymbol{b}}_i & \vec{\hat{\alpha}}_i \geq \rho_d, \\ 0 & \text{otherwise} \end{cases} \forall i$$

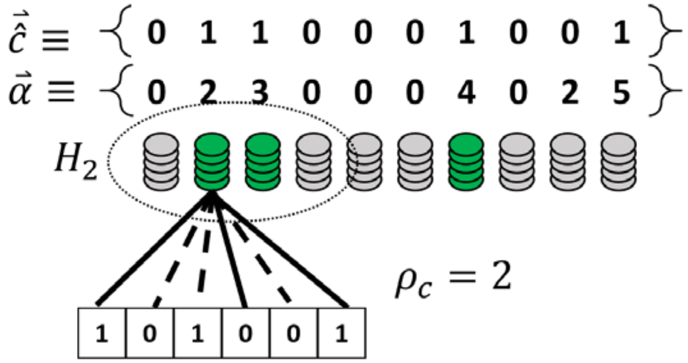
$$\vec{\hat{\alpha}}_i \equiv \boldsymbol{X}_i \bullet \boldsymbol{Y}_i$$



Inhibition

$$\vec{\boldsymbol{c}} \equiv \mathrm{I}(\vec{\boldsymbol{\alpha}}_i \geq \vec{\boldsymbol{\gamma}}_i)\ \forall i$$

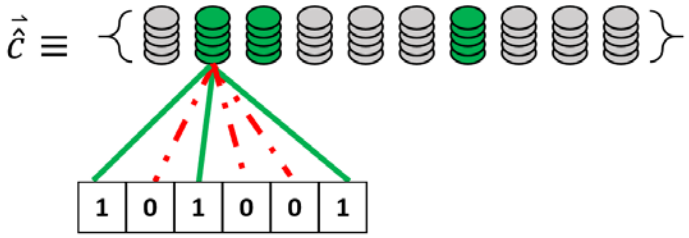
$$\vec{\boldsymbol{\gamma}} \equiv \max(\mathrm{kmax}(\boldsymbol{H}_i \odot \vec{\boldsymbol{\alpha}}, \rho_c), 1)\ \forall i$$



Learning

$$\boldsymbol{\Phi} \equiv \mathrm{clip}(\boldsymbol{\Phi} \oplus \boldsymbol{\delta\Phi}, 0, 1)$$

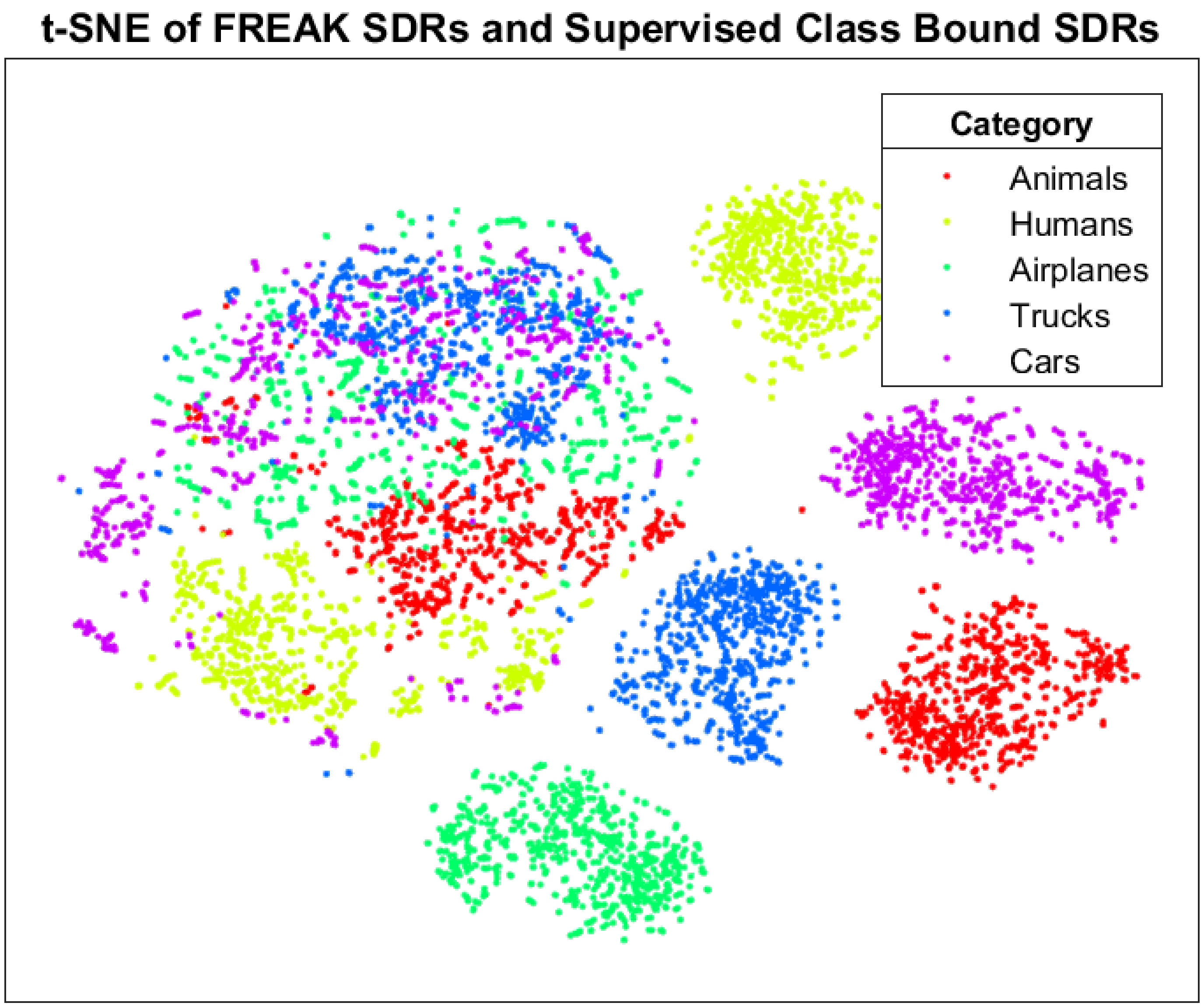
$$\boldsymbol{\delta\Phi} \equiv \vec{\hat{\boldsymbol{c}}}^T \odot (\phi_+ \boldsymbol{X} - (\phi_- \neg \boldsymbol{X}))$$



Mnatzaganian, James, Ernest Fokoué, and Dhireesha Kudithipudi. "A mathematical formalization of hierarchical temporal memory's spatial pooler." *Frontiers in Robotics and AI* 3 (2017): 81.

Motivation

- Numenta's research is focused on bioplausibility and emulation of the pathways in the Neocortex
- Binding operation is the basis for *Content Addressable Memory*, which could help facilitate long term storage and retrieval of SDRs
- Combine multimodal data without increasing dimensionality



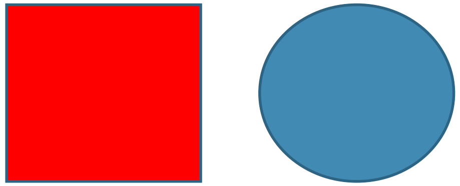
Background on Binding

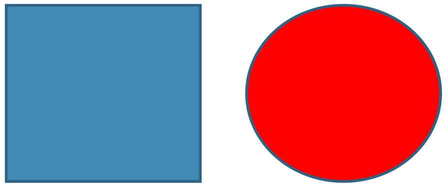
Binding problem for language and vision (Jackendoff 2003, Malsburg 1995)

- Vector Symbolic Architectures (Proposed by Gayler 2004)
- Vector operations (Binding, Superposition, and Permutation)
- Vectors are the same dimensionality; both atomic and complex representations

Different Implementations of VSA

- Holographic Reduced Representations (HRR)- Plate
- Binary Spatter Codes / Hyperdimensional Computing – Kanerva
- Multiply, Add, Permute (MAP) – Gayler

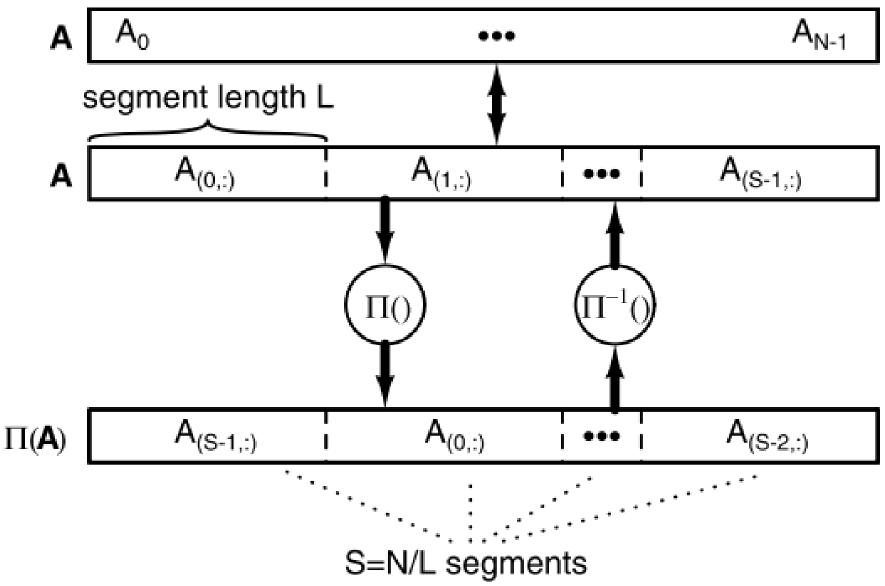




Binding – Binary Vectors

Sparse Binary Distributed Representations

- (Laiho & Kanerva 2015) proposed a binding operation for sparse binary distributed representations
- Segmentation of sparse binary vector
- *Maximally Sparse* – requires controlled density
- Generalization of circular convolution for *Plate's* HRRs in the frequency domain.



Laiho, M.; Poikonen, J. H.; Kanerva, P.; and Lehtonen, E. 2015. High-dimensional computing with sparse vectors. In Biomedical Circuits and Systems Conference (BioCAS), 2015 IEEE, 1–4. IEEE.

Binding

C = A $\otimes$ B

N = 20
L = 5
S = N/L

A	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0
j =	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
B	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0
j =	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4
C	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0

$$a_s = \min(\arg\max_j(\mathbf{A}_{(s,j)}))$$

$$\mathbf{C}_{(s,j)} :\equiv \mathbf{B}_{(s,j-\alpha a_s\ (L))}.$$

Unbinding

$$B = A \oslash C$$

N = 20
L = 5
S = N/L

A	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0
j =	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4

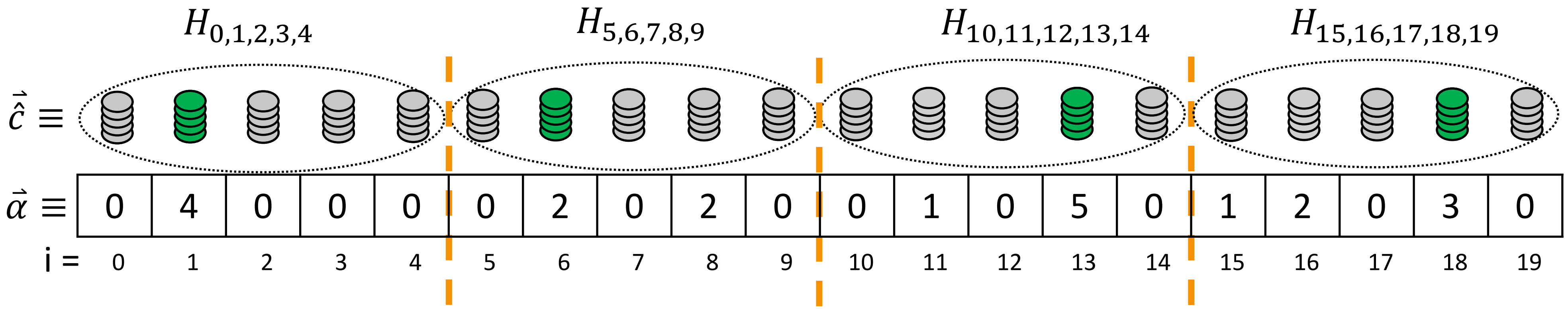
C	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0
j =	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4

B	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0

Spatial Pooler – Maximally Sparse SDRs

Use Local Inhibition in the Spatial Pooler Algorithm

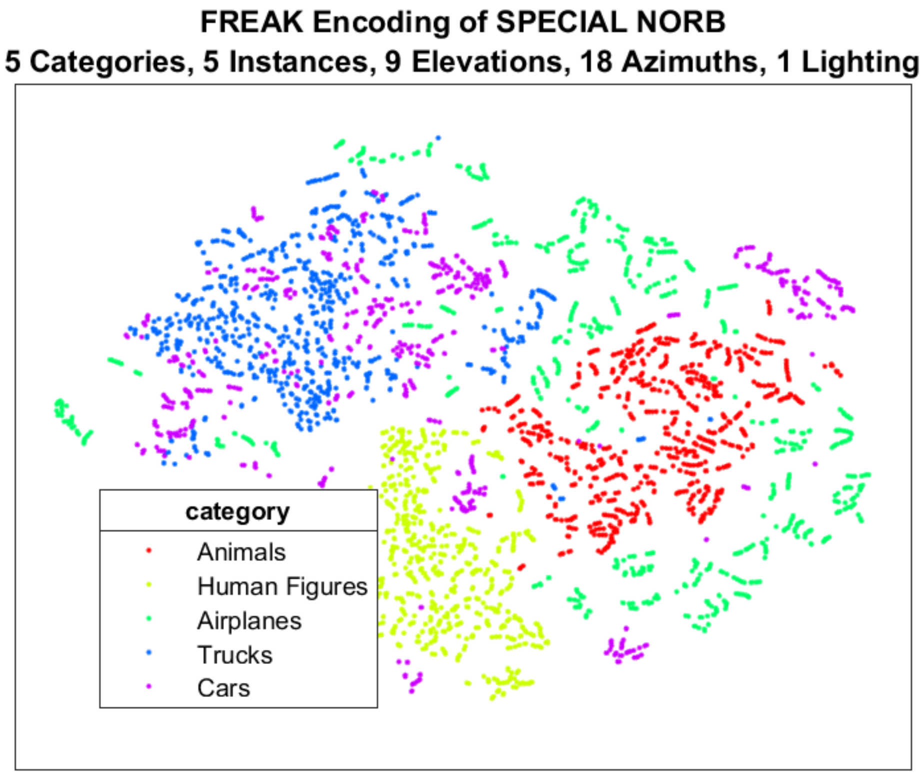
- Use neighborhood masks, *H*, to establish segments in the HTM Region
- Select the most active column in each neighborhood/segment instead of top *k* columns
- Initialization of column to input connectivity is based on the dataset. Requires a mapping that preserves the distributed nature (topology) of SDRs



FREAK – Fast Retina Keypoint

FREAK Encoding

- 512 bit binary descriptor
- Difference of Gaussians (No Backprop!)
- Coarse to fine grained information
- Inspired by the retina, but no biological significance

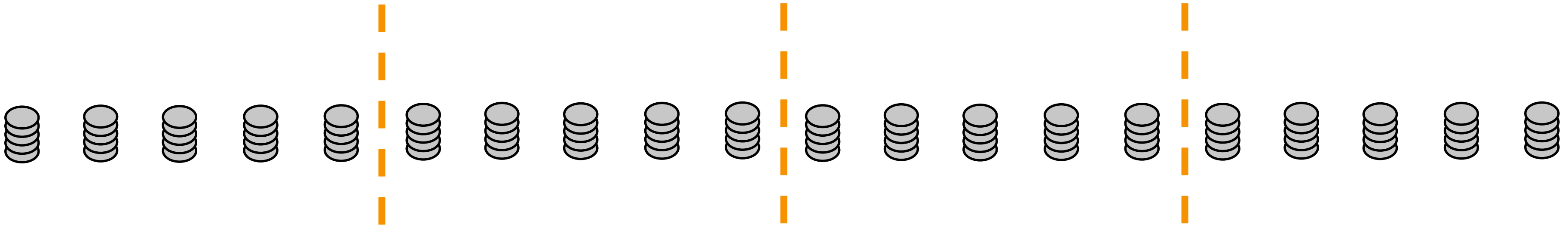


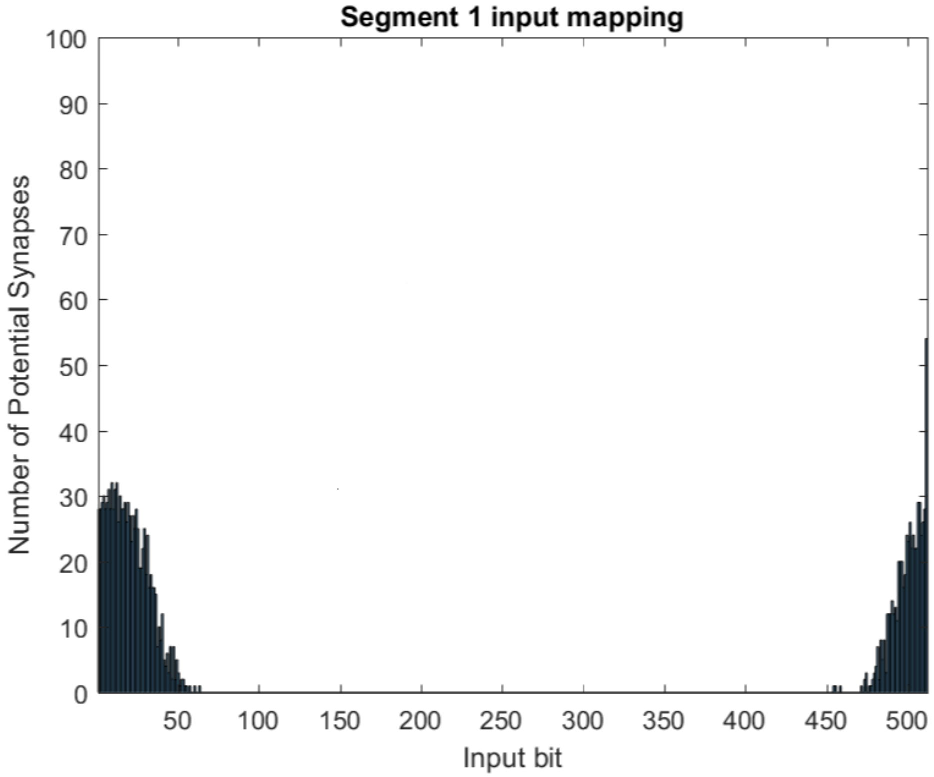
Encoding small NORB

Alahi, A.; Ortiz, R.; and Vandergheynst, P. 2012. FREAK: Fast Retina Keypoint. In 2012 IEEE Conference on Computer Vision and Pattern Recognition, 510–517. IEEE

Spatial Pooler – Initialization

HTM Columns





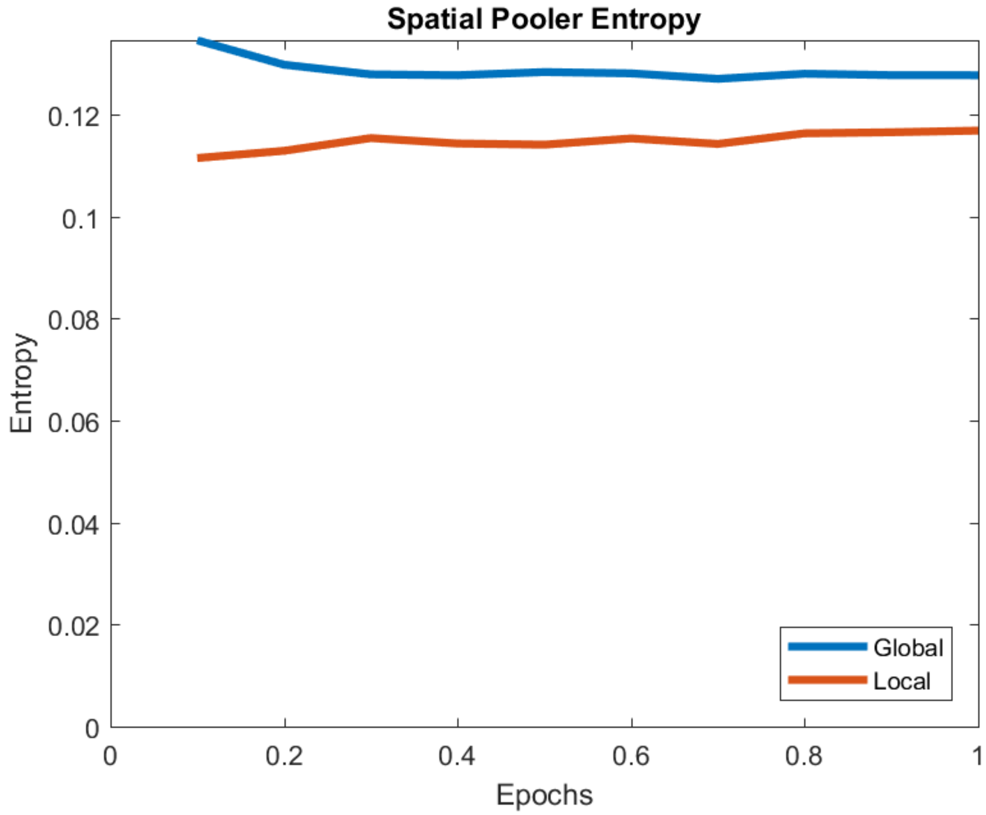
FREAK Descriptor

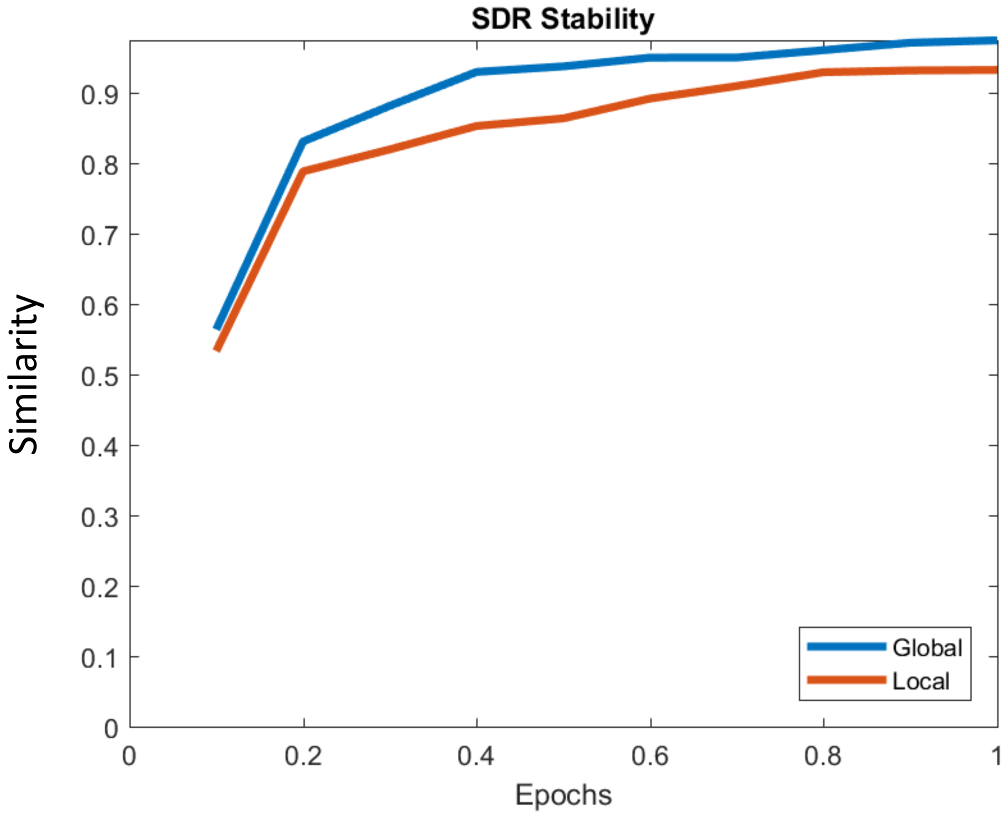
Coarse → **Fine**

Significant Hyperparameters:

- 2048 columns (mini-columns)
- 64 active bits (~3% Sparsity)
- 1 epoch of learning on NORB training set

Spatial Pooler – Learning Metrics

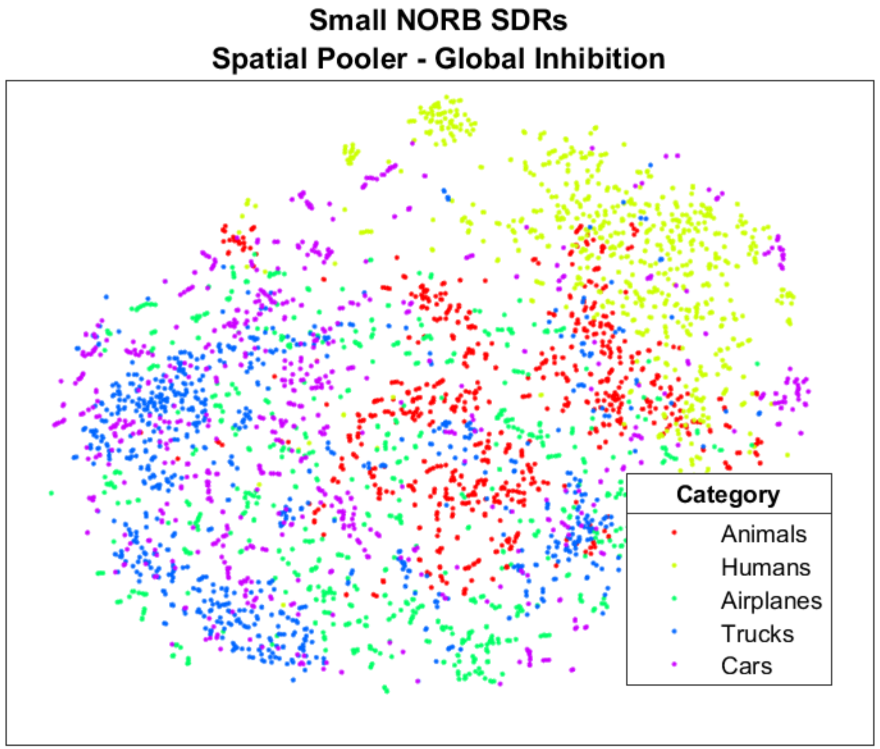


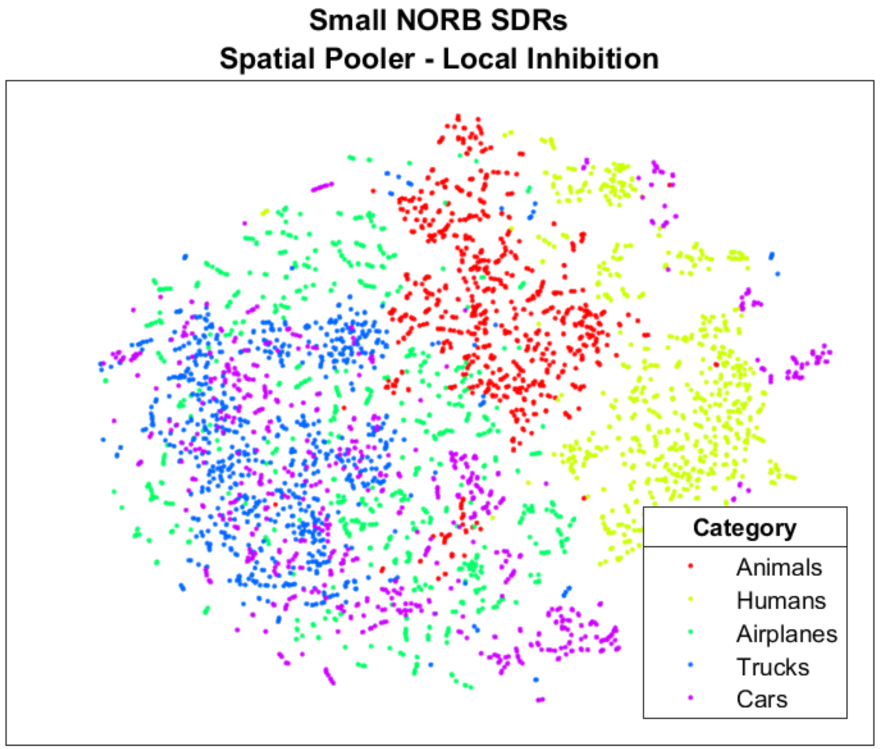


Spatial Pooler performance while training:

- Local Inhibition has less column participation because of the stricter inhibition rule
- Local Inhibition doesn’t fully stabilize after one epoch because of random selection between columns with equal overlap/activation levels.

Spatial Pooler – Cluster Analysis with t-SNE





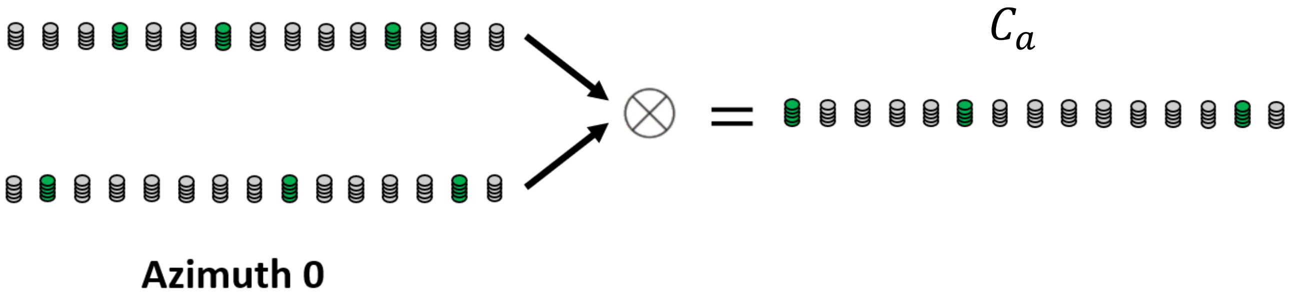
Entangled Nature of SDRs:

- Global Inhibition produces more entangled representations than the raw FREAK Descriptors
- Local Inhibition helps preserve more of the class similarity (i.e. larger and identifiable clusters)

Binding Experiments

Binding Location & Feature SDRs

- Bind each features SDR with a location SDR (Supervised)
- Randomly generate 18 *maximally sparse* SDRs for each possible azimuth

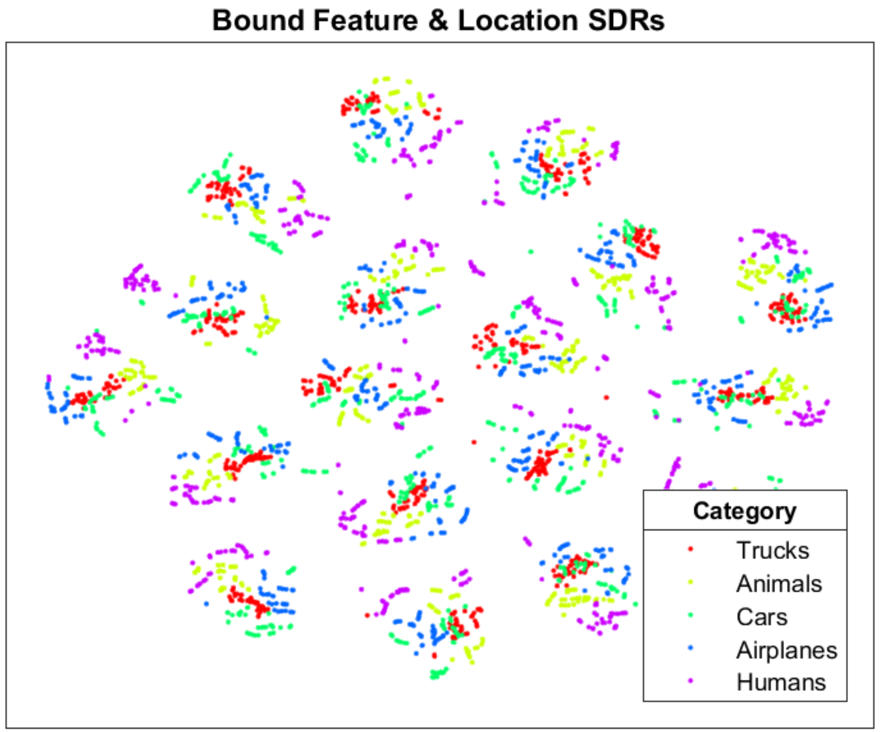


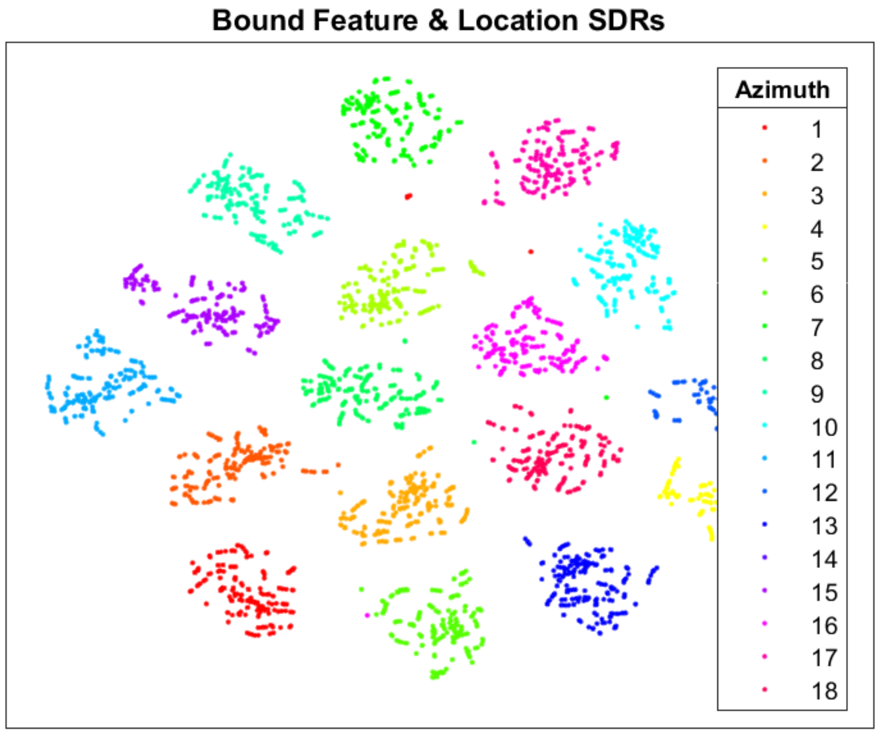
Binding & Superposition

- Integrate the bound representation as a rotation around the object (repeat for each elevation)
- Superposition of SDRs (Logical **OR**)

$$Rotation\ SDR = \sum_{a=0}^{17} C_a$$

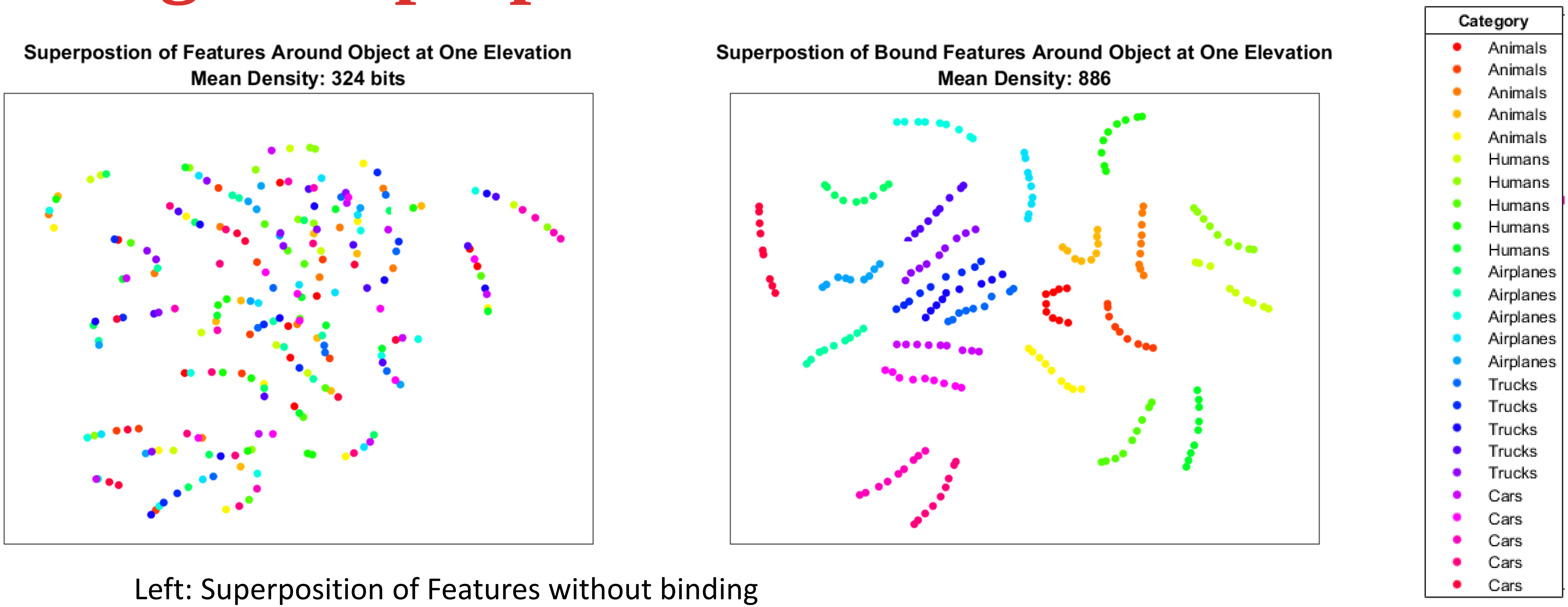
Binding Feature & Location SDRs





- Same Dimensionality and Sparseness as component SDRs
- These SDRs are Content Addressable:
  - Unbind with an azimuth SDR => “What feature(s) is located at this Azimuth?”
  - Unbind with a feature SDR => “What azimuth(s) is associated with this feature?”

Binding & Superposition



Left: Superposition of Features without binding

- Larger density (~16% active bits)
- SDRs from different class/instances are similar (more overlapping bits)

Right: Superposition of Features bound with corresponding azimuth.

- Much larger density (~50% active bits)
- Binding gives more structure to superimposed representations

Conclusions

Basis for Content Addressable Memory

- Binding can give structure to entangled representations, but requires a supervised approach
- Density of superimposed representations will be a limiting factor for recall or retrieving vectors from superimposed SDRs
- Novel vector, non-similar to either component

Local Inhibition

- Minor modification to inhibition in Spatial Pooler Algorithm