

# Braindrop: A Mixed-Signal Neuromorphic System that Presents Clean Abstractions

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# Deep learning is huge —in the cloud

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- ❖ Backprop learning is powerful
  - ❖ Networks deep in space or time
  - ❖ Space is discretized into layers
  - ❖ Time is discretized into steps
- ❖ Unit's output must be differentiable (with respect to outputs of units feeding it)

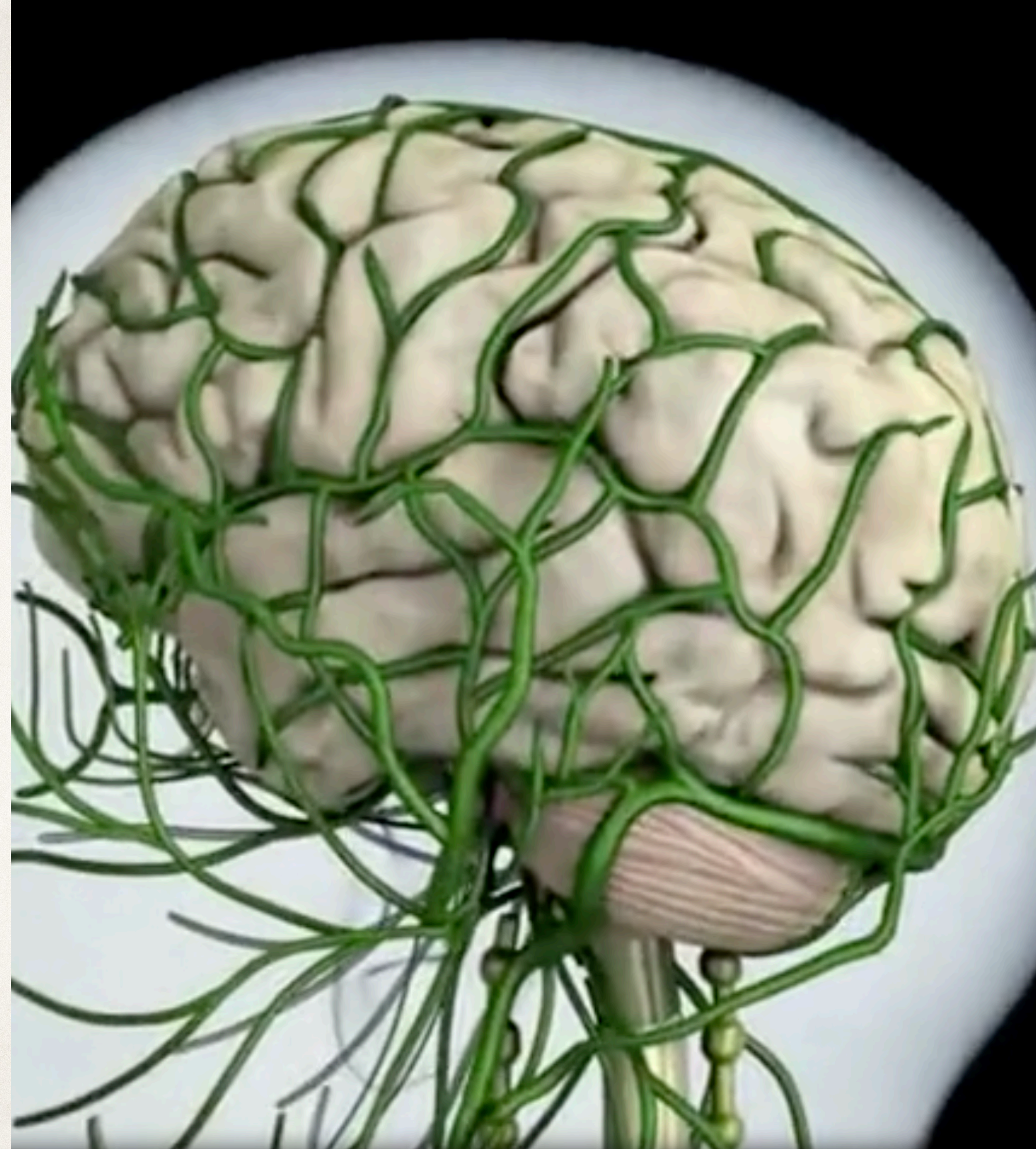




# Backprop's constraints limit design-space

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- ❖ Cannot take advantage of:
  - ❖ Physical space (its continuous)
  - ❖ Real time (its also continuous)
  - ❖ Non-differentiable signals (e.g., spikes)





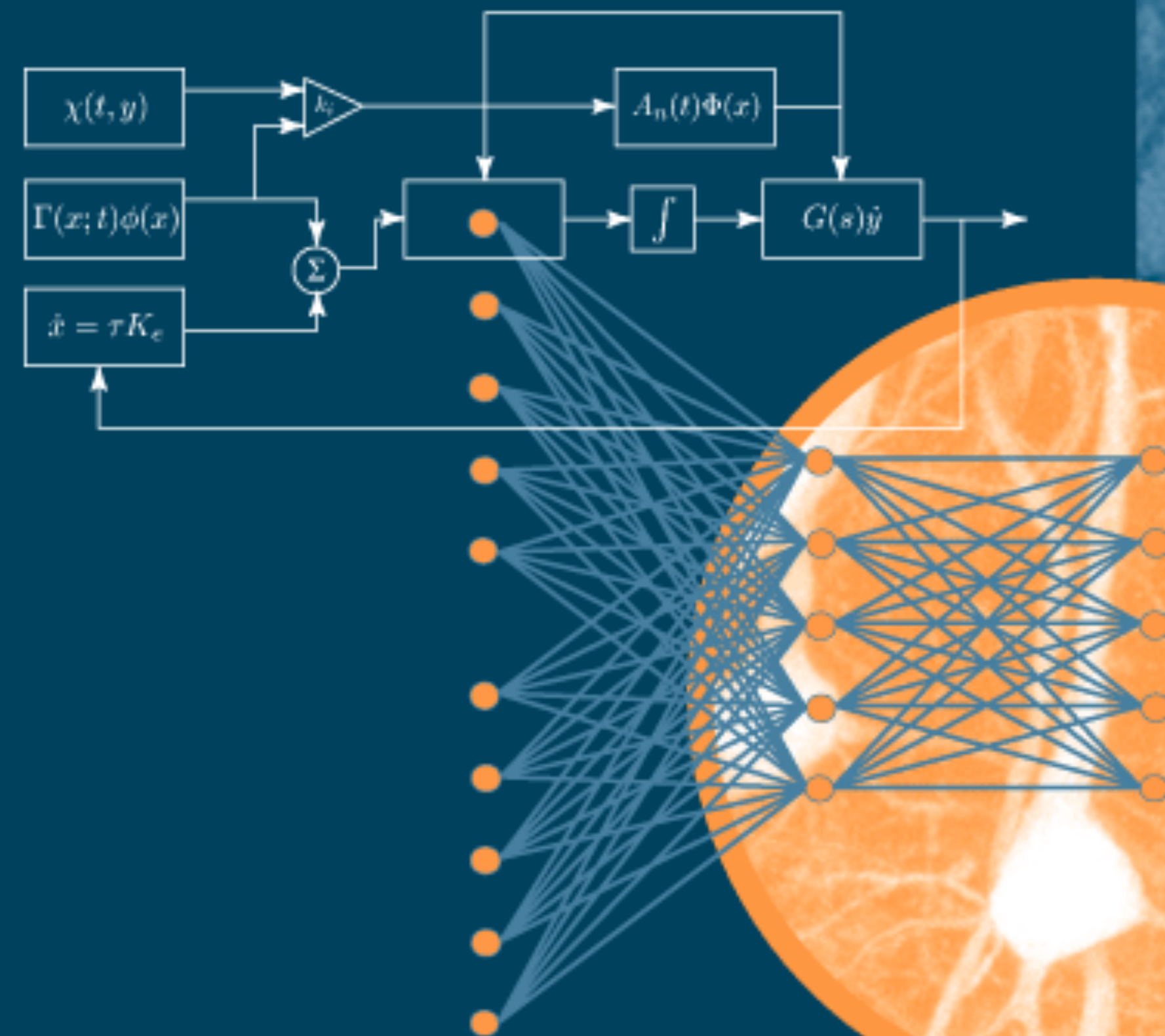
# How do we relax its constraints? (Part I)

- ❖ Map functional abstractions onto physical ones
- ❖ Two existing examples:
  - ❖ Neural Engineering Framework (Eliasmith & Anderson 2003)
  - ❖ Predictive Coding Framework (Deneve et al. 2014)

## Neural Engineering

COMPUTATION, REPRESENTATION, AND DYNAMICS  
IN NEUROBIOLOGICAL SYSTEMS

Chris Eliasmith and Charles H. Anderson

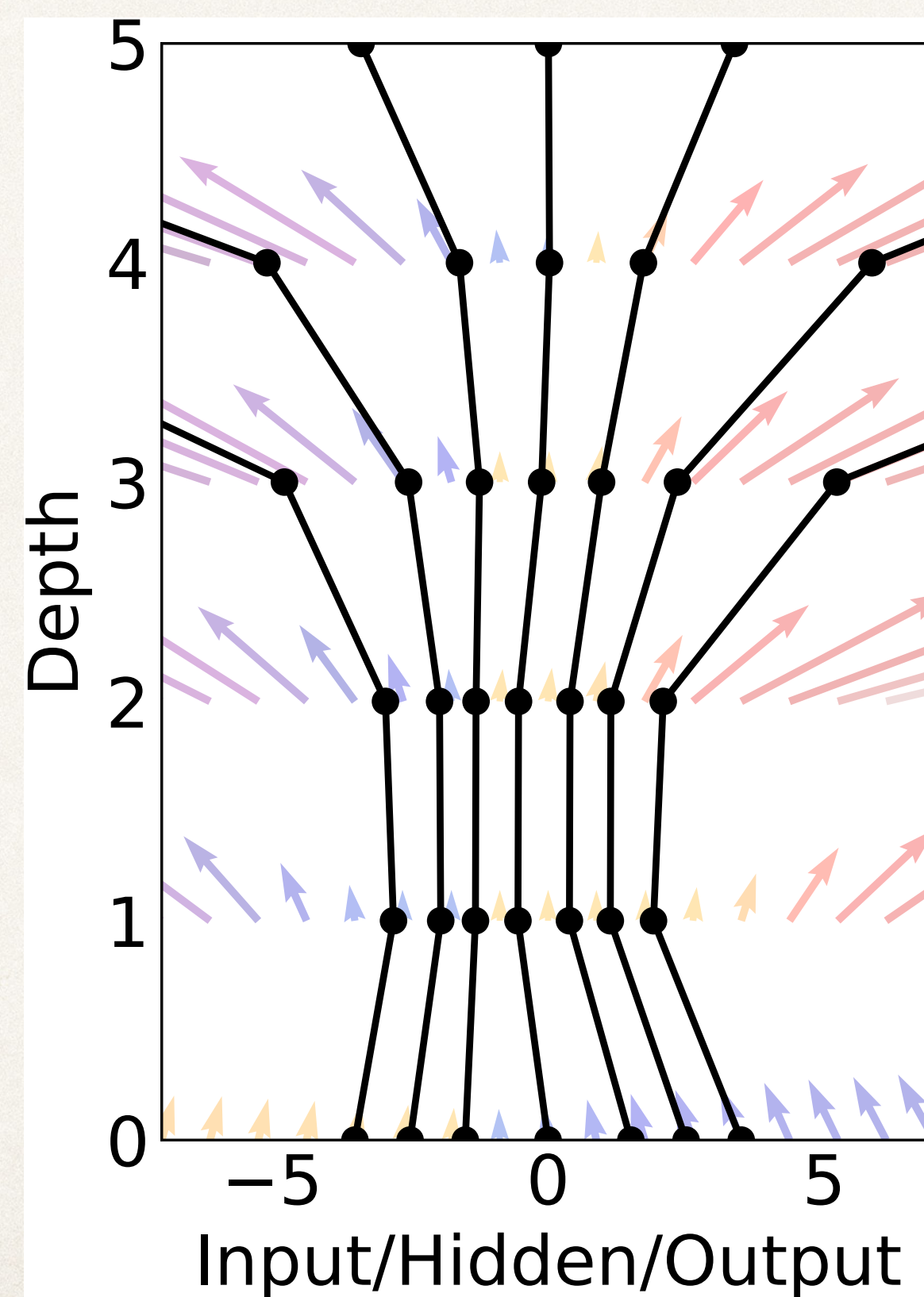




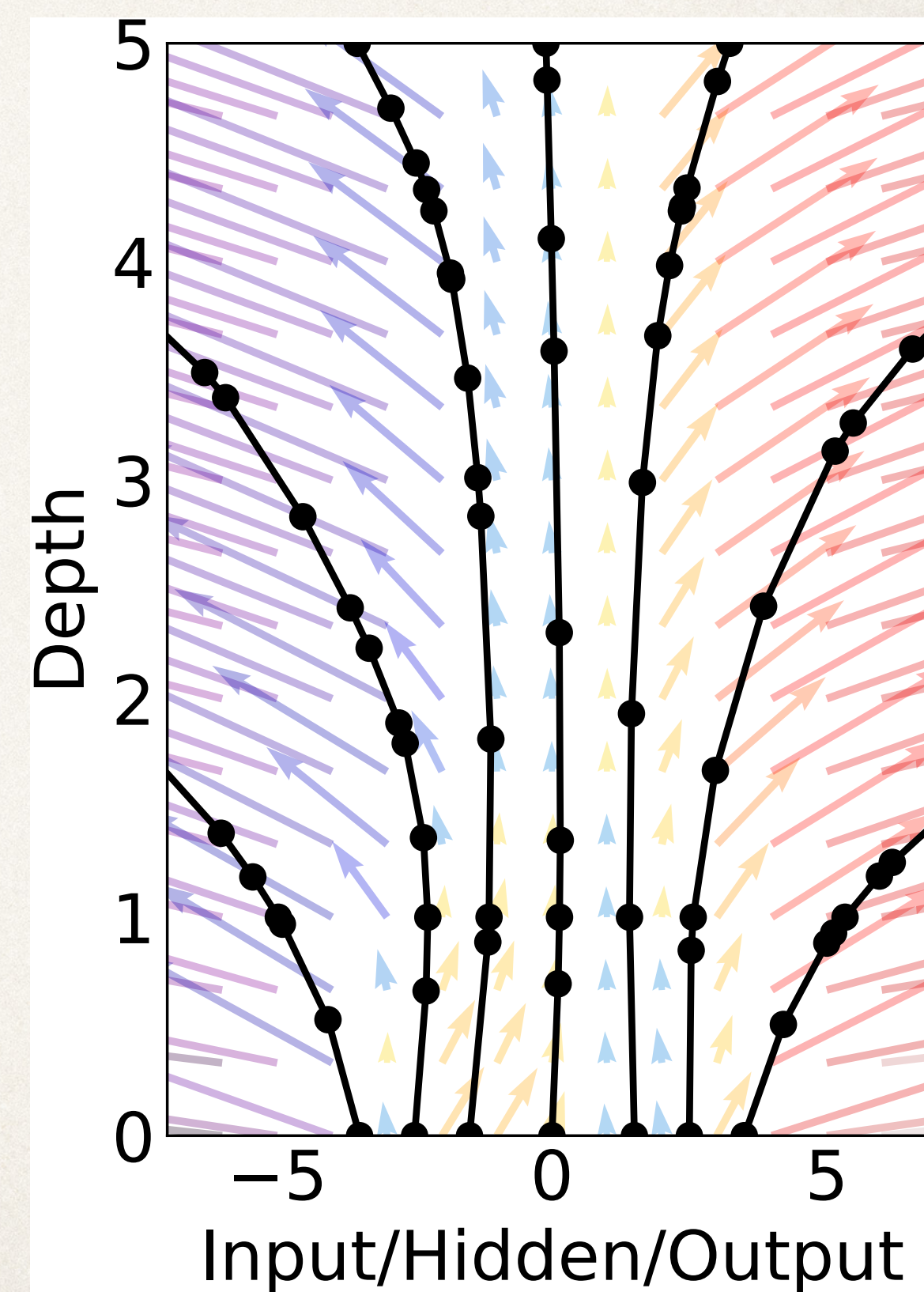
# How do we relax its constraints? (Part II)

- ❖ Train networks continuous in time and space
- ❖ Known as dynamical systems
- ❖ An existing example:
  - ❖ Neural Ordinary Differential Equations (Duvenaud et al. 2018)

Residual Network

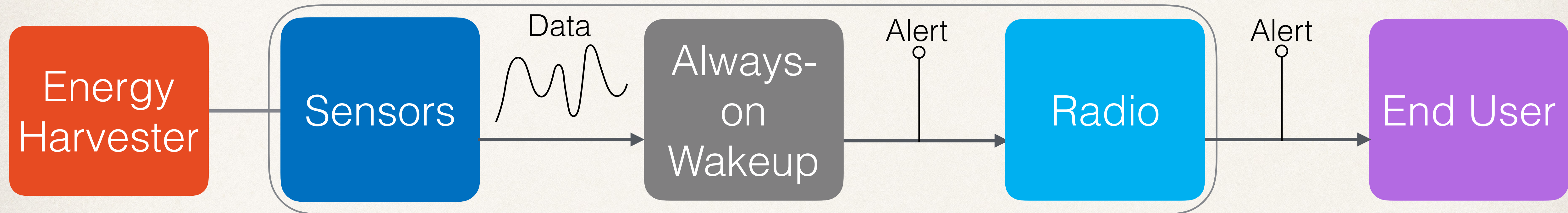


ODE Network





# What's the payoff? Learning at the edge



- ❖ Exploit physical primitives to implement physical abstractions
- ❖ Reap dramatic gains in energy-efficiency



**Harvest**  
Vibration: 500  $\mu\text{W}$



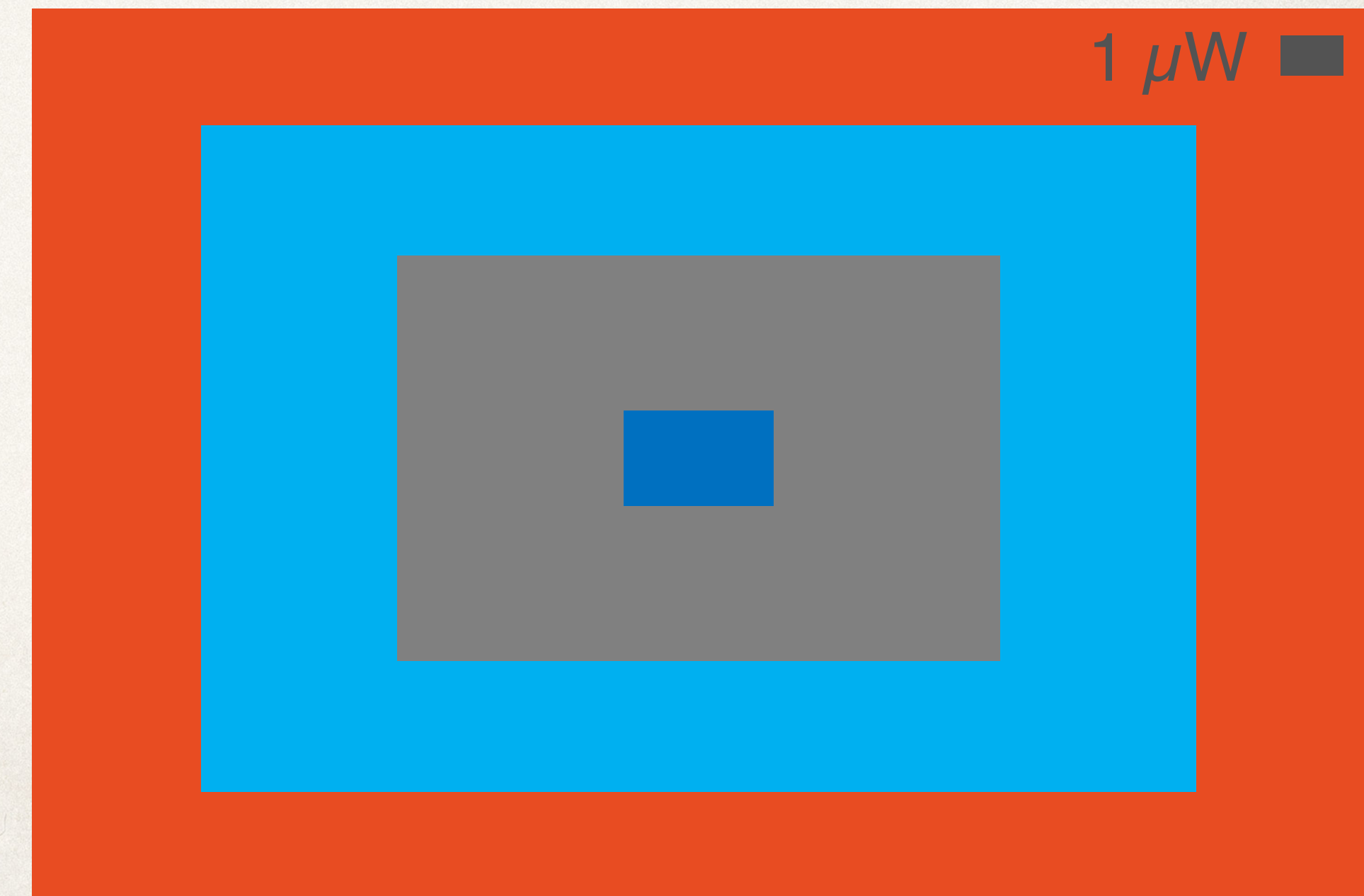
**Sense**  
Accelerometer: 6  $\mu\text{W}$



**Compute**  
Neuroprocessor: 100  $\mu\text{W}$



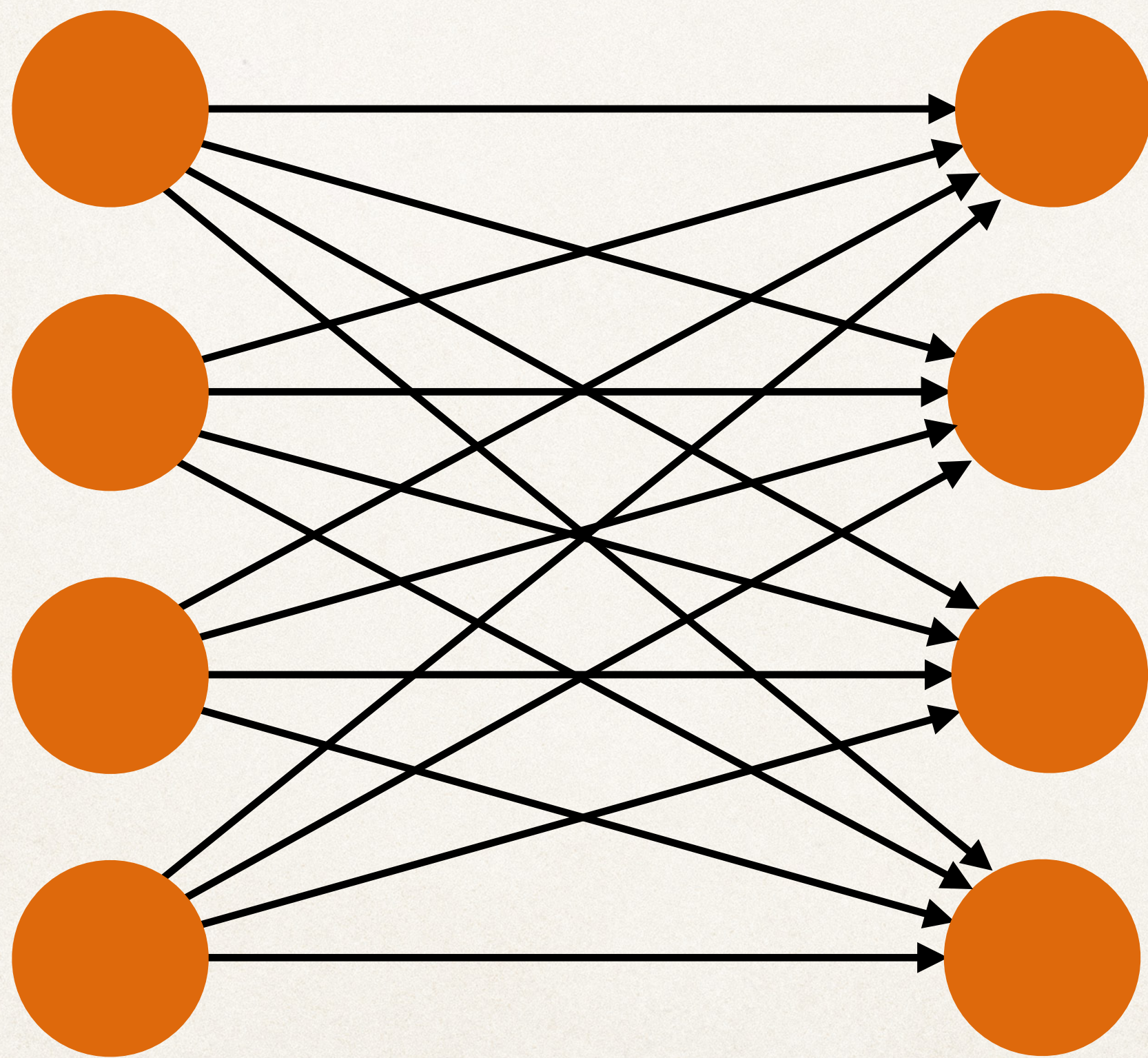
**Communicate**  
BTLE (1% on-time): 280  $\mu\text{W}$





# Minimizing energy

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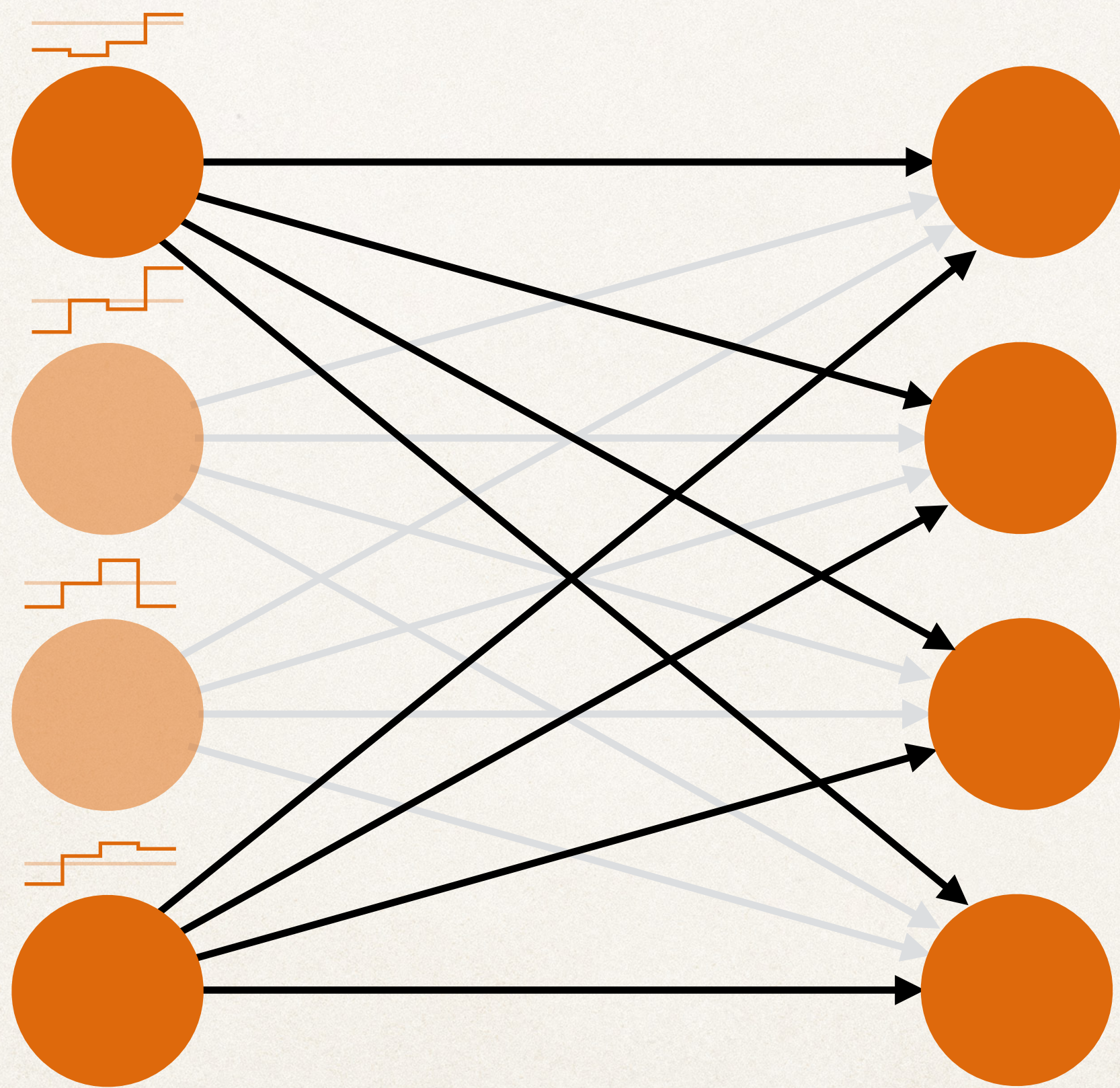


$$\begin{aligned} E_{op} &= N_{active} N_{conn} \\ &= (\rho_{active} N)(\rho_{conn} N) \end{aligned}$$



# Minimizing energy: Temporal sparsity

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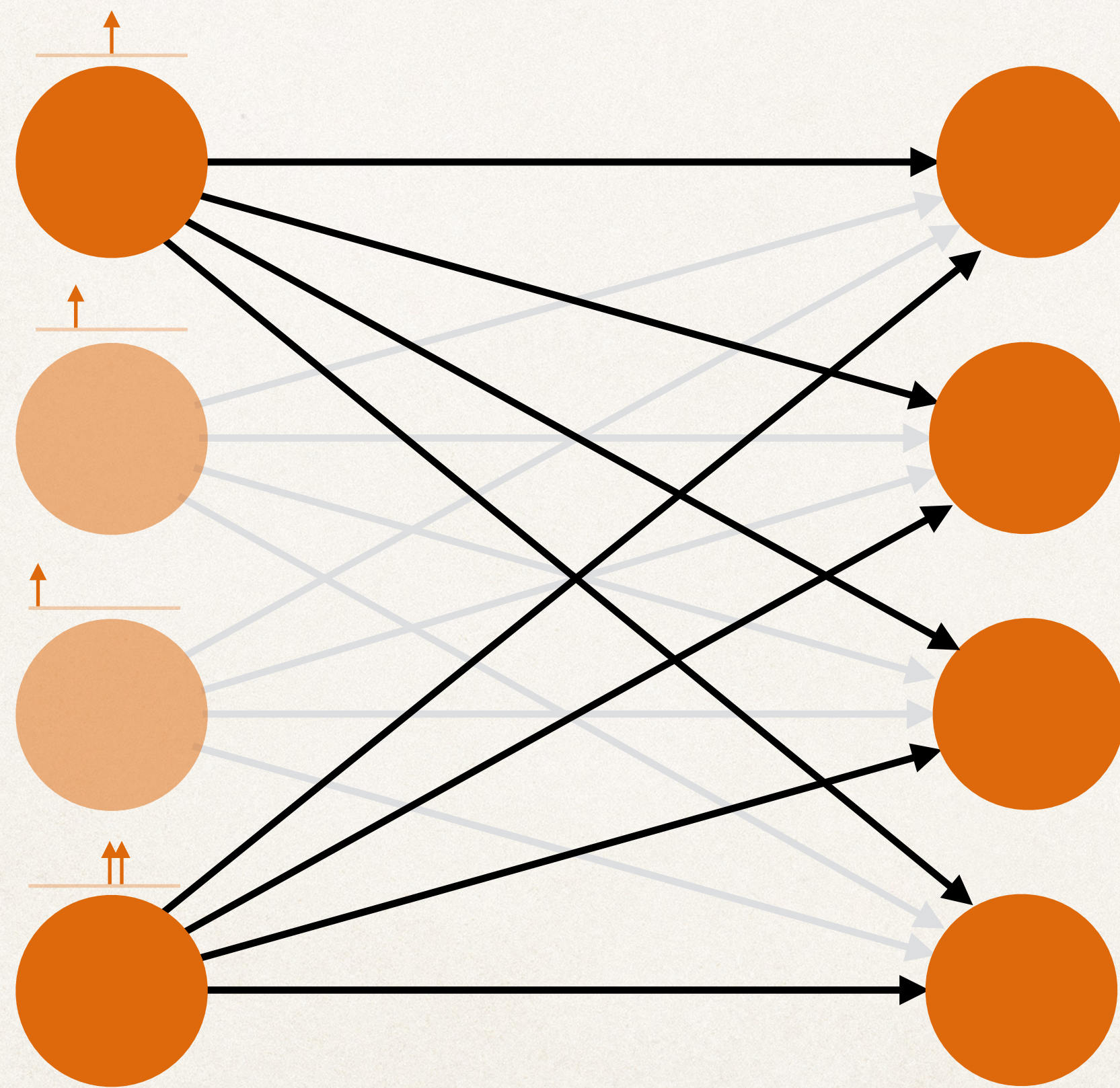


$$\begin{aligned} E_{op} &= N_{active} N_{conn} \\ &= (\rho_{active} N)(\rho_{conn} N) \end{aligned}$$



# Temporal sparsity: Spikes

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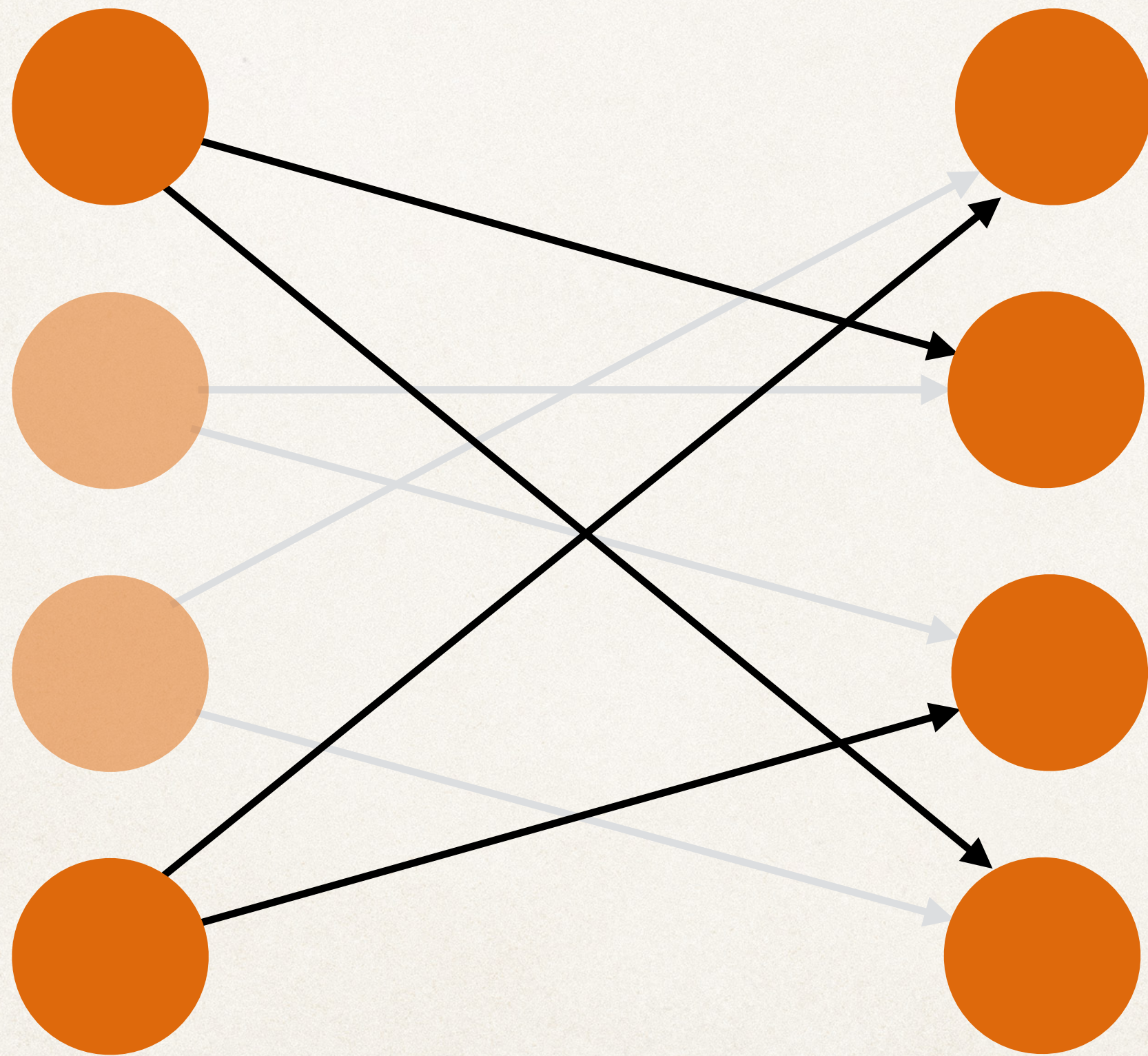


$$\begin{aligned} E_{op} &= N_{active} N_{conn} \\ &= (\rho_{active} N)(\rho_{conn} N) \end{aligned}$$



# Minimizing energy: Spatial sparsity

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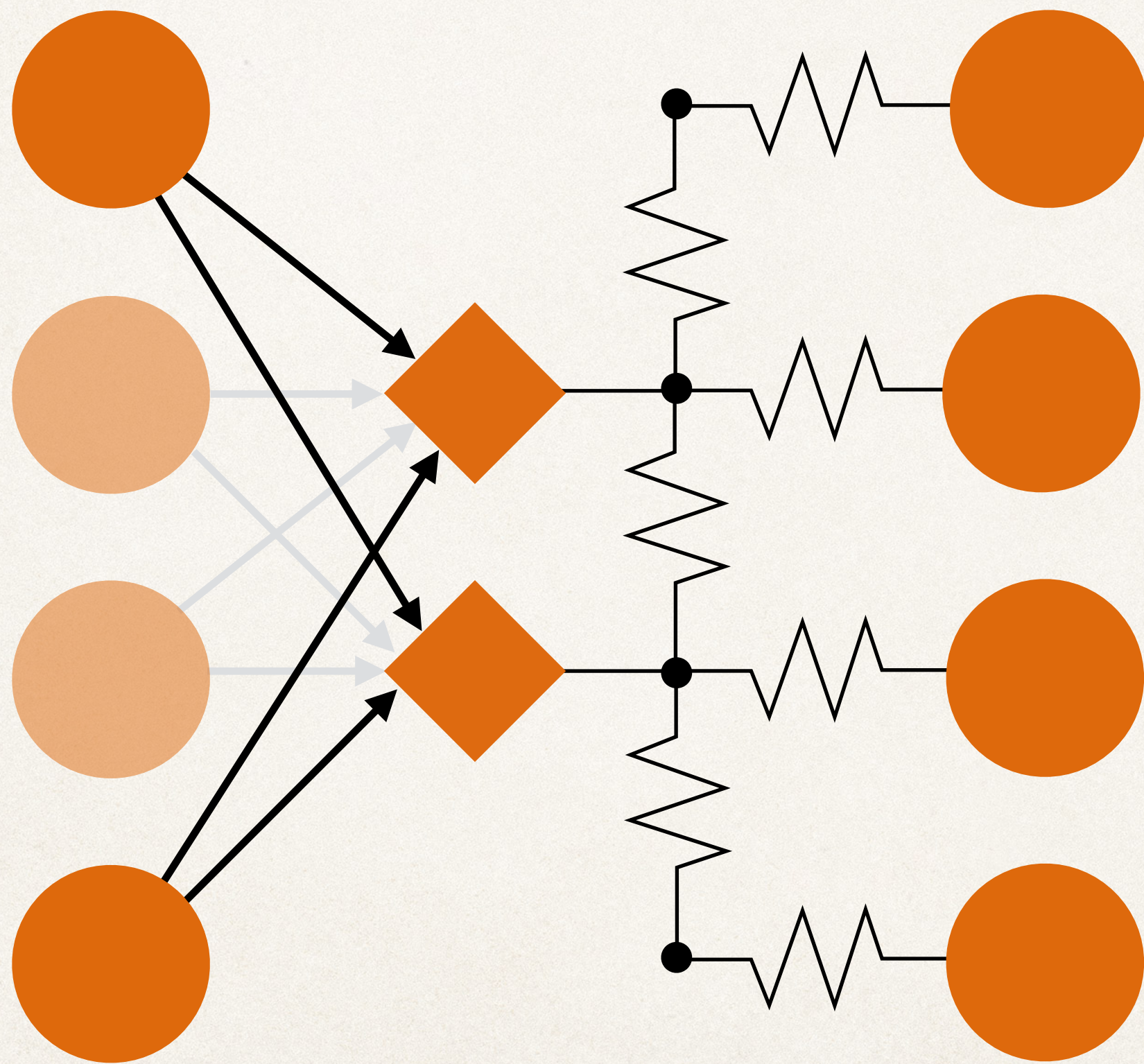


$$\begin{aligned} E_{op} &= N_{active} N_{conn} \\ &= (\rho_{active} N) (\rho_{conn} N) \end{aligned}$$



# Spatial sparsity: Analog convolving

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$$\begin{aligned} E_{op} &= N_{active} N_{conn} \\ &= (\rho_{active} N) (\rho_{conn} N) \end{aligned}$$



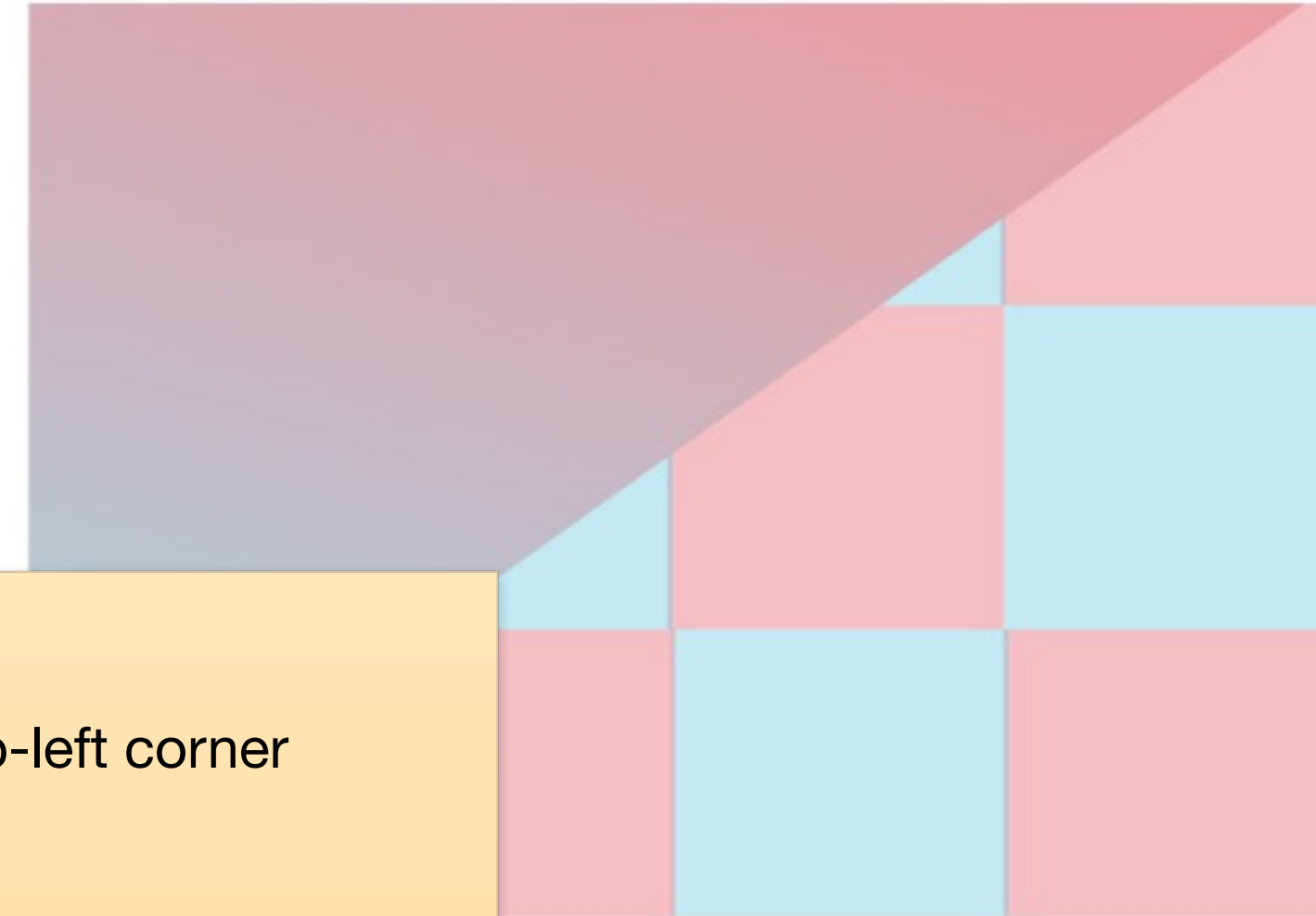
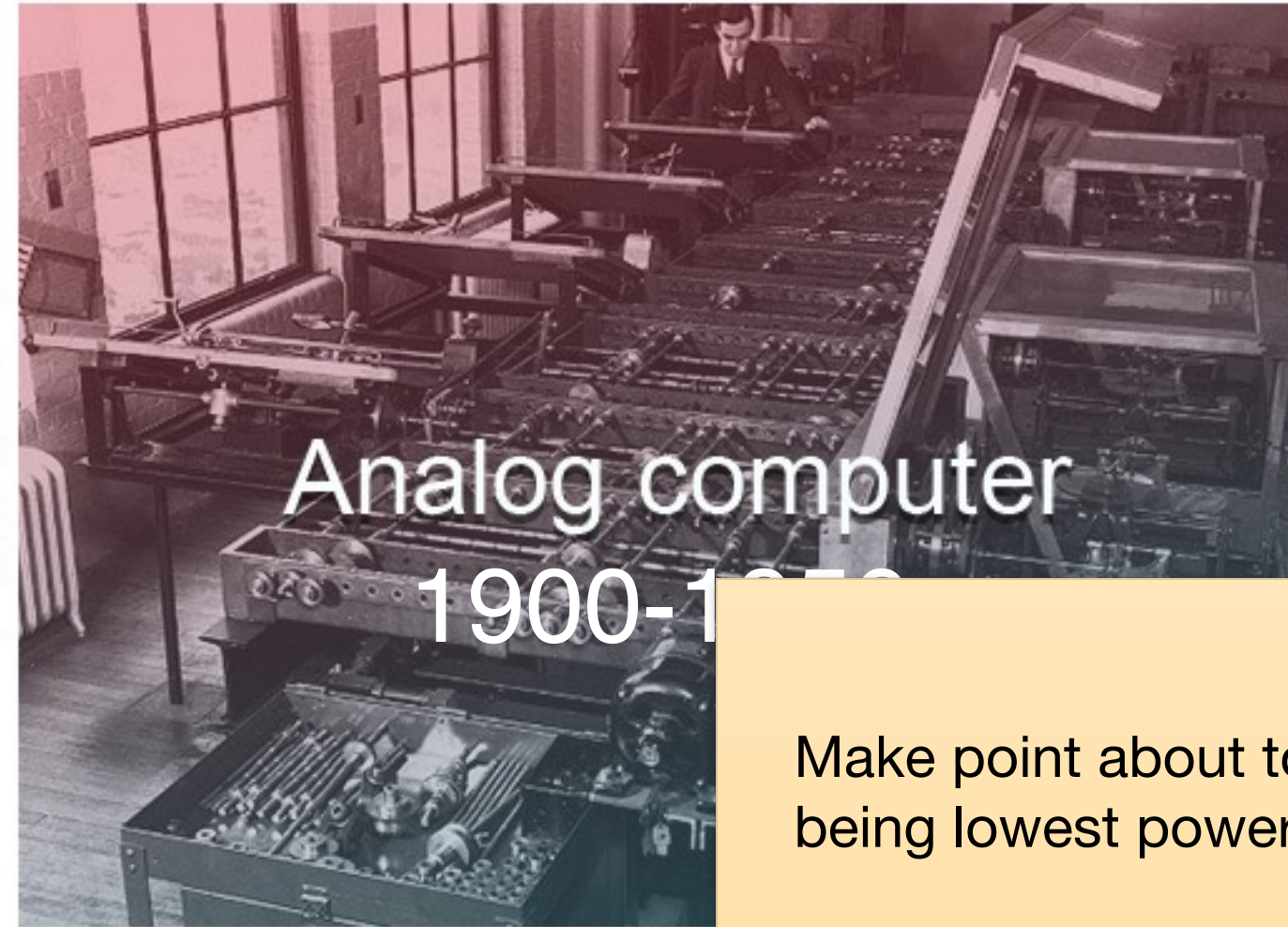
# COMPUTATION

ANALOG

DIGITAL

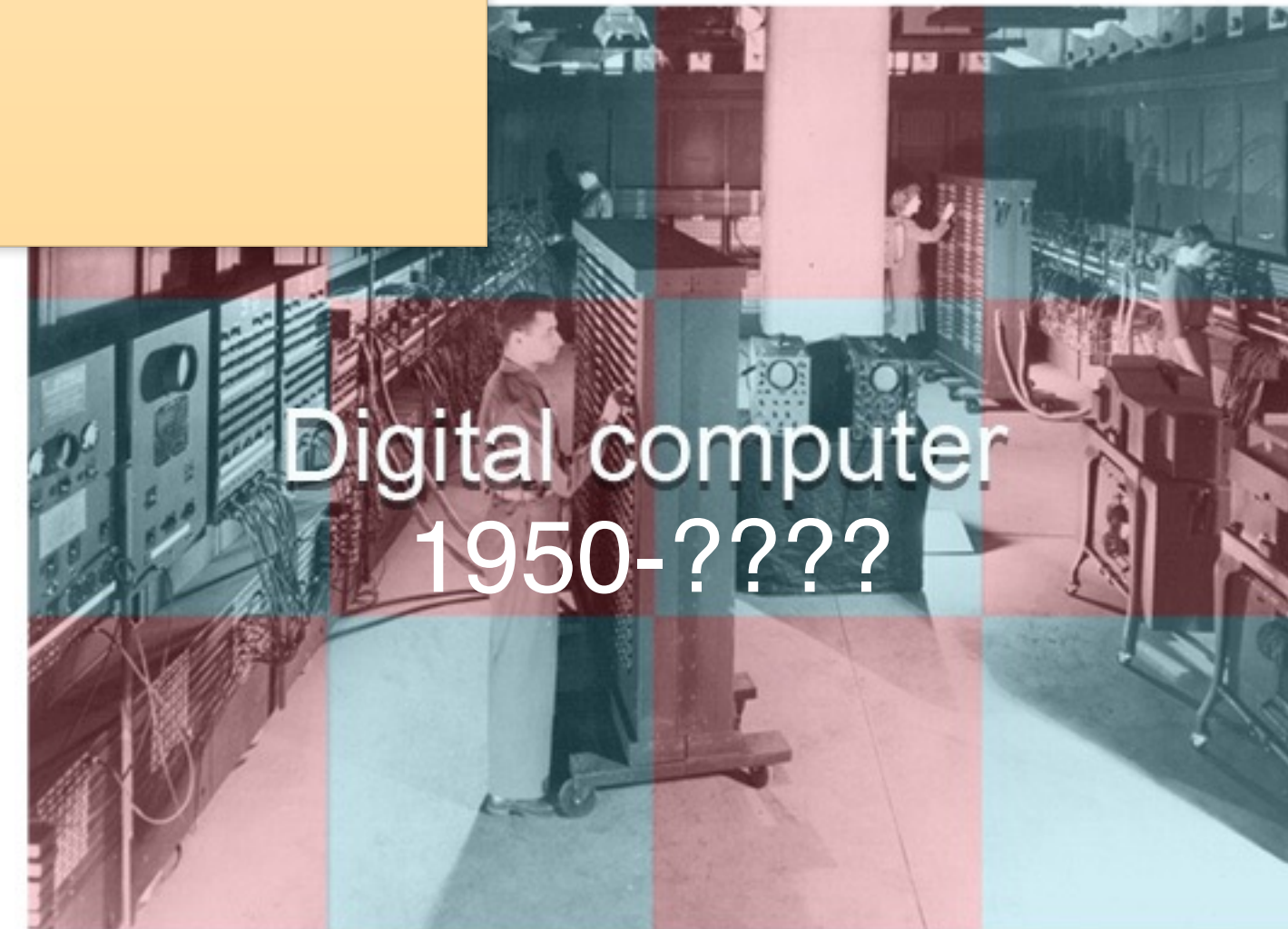
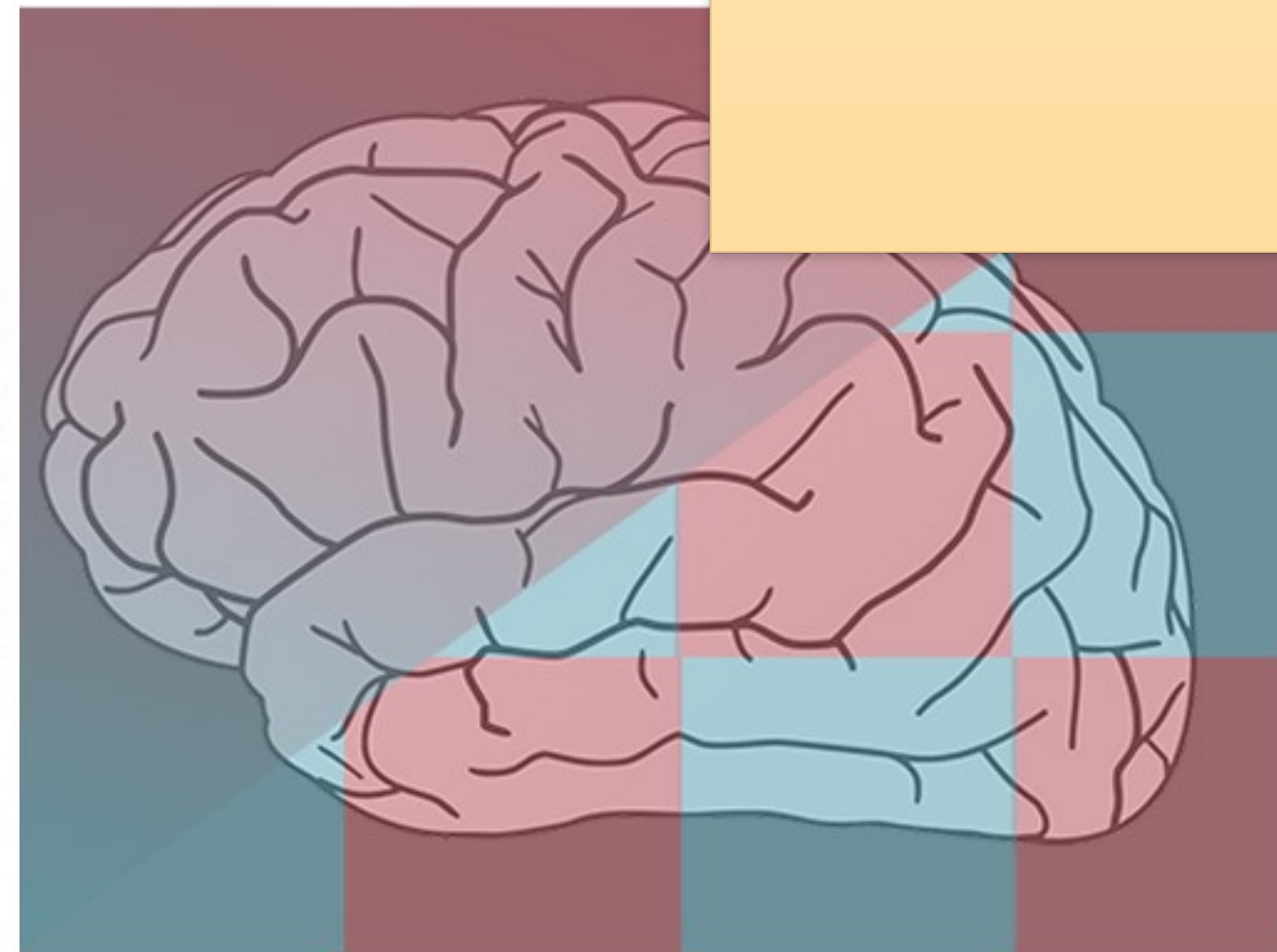
COMMUNICATION

ANALOG



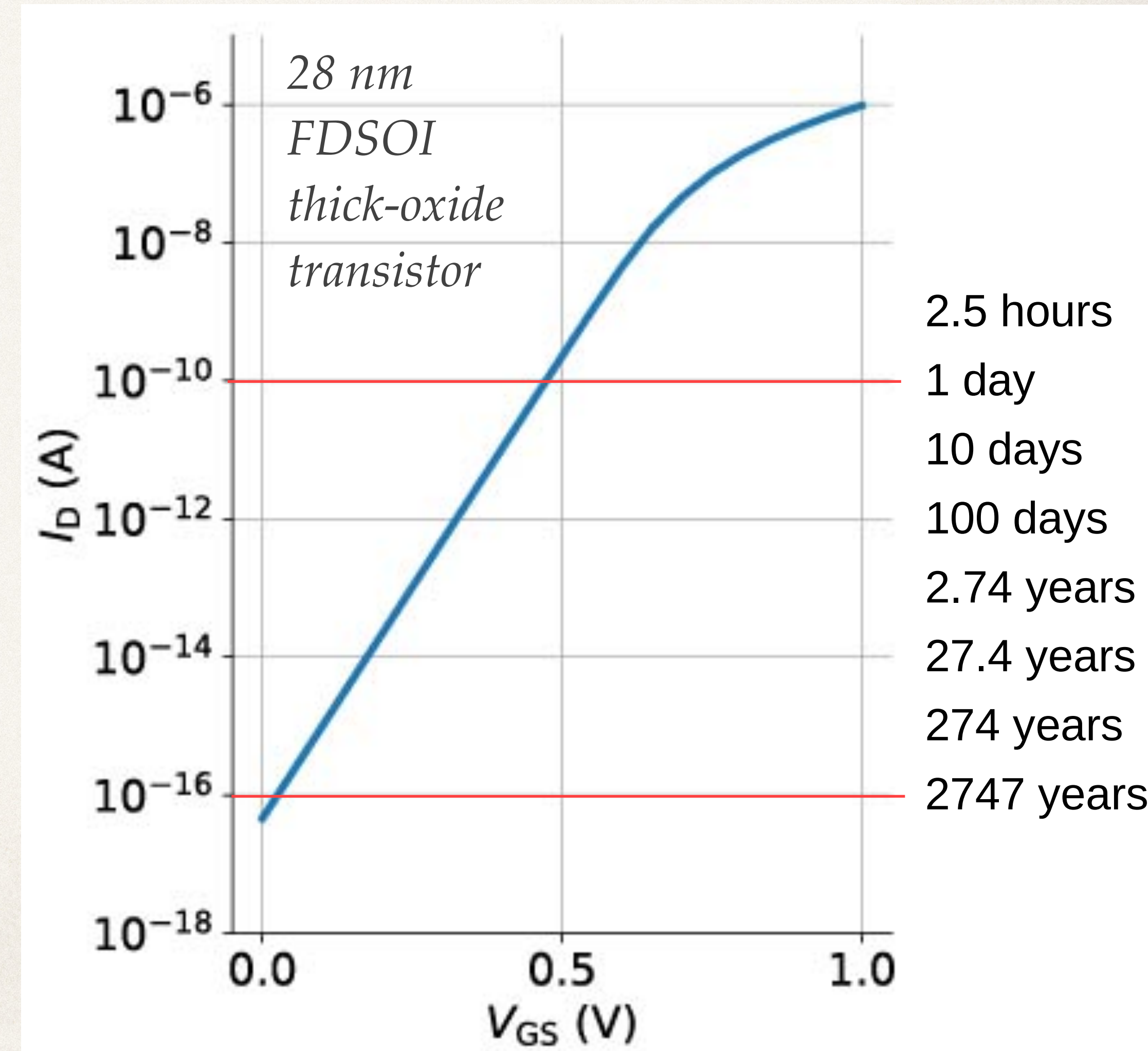
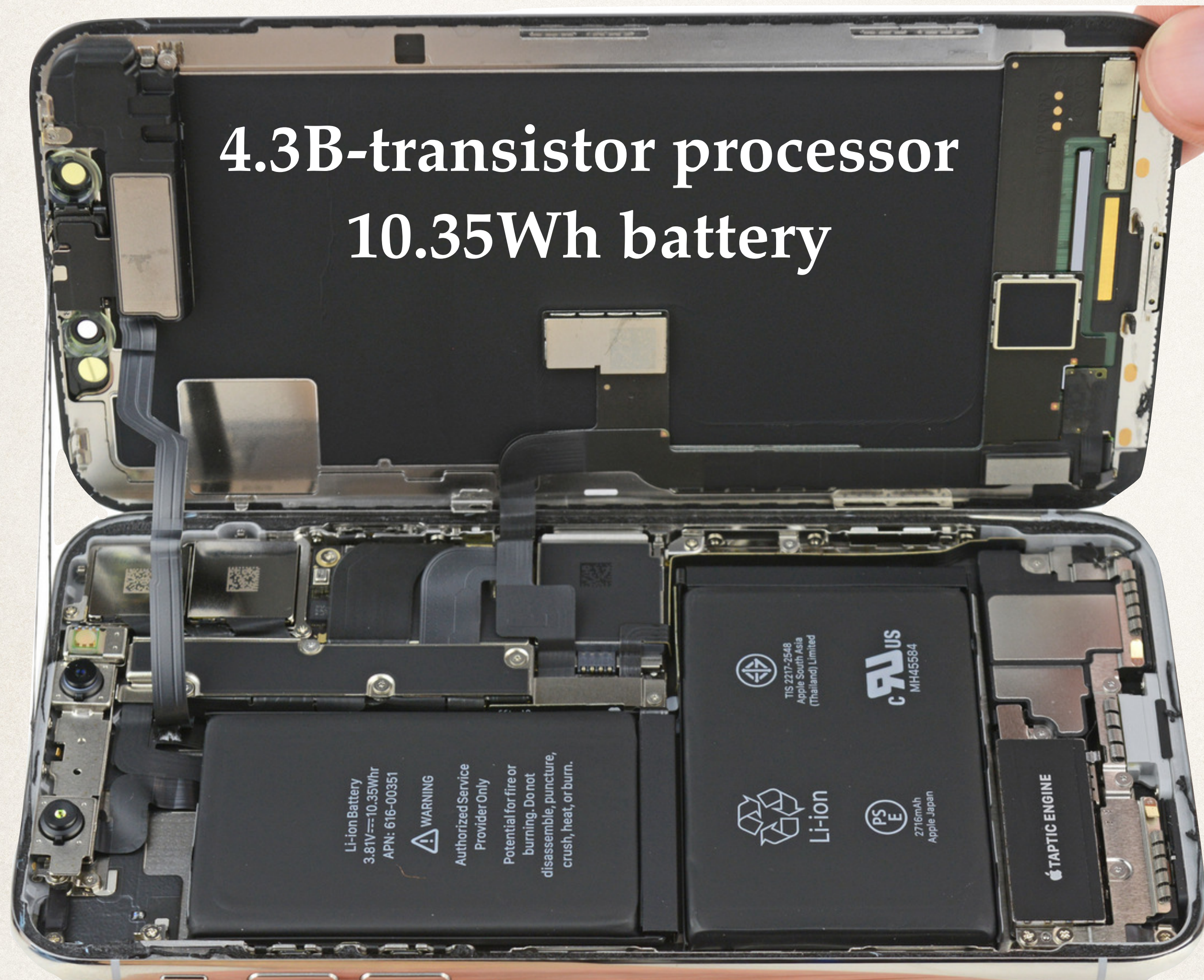
Make point about top-left corner being lowest power

DIGITAL

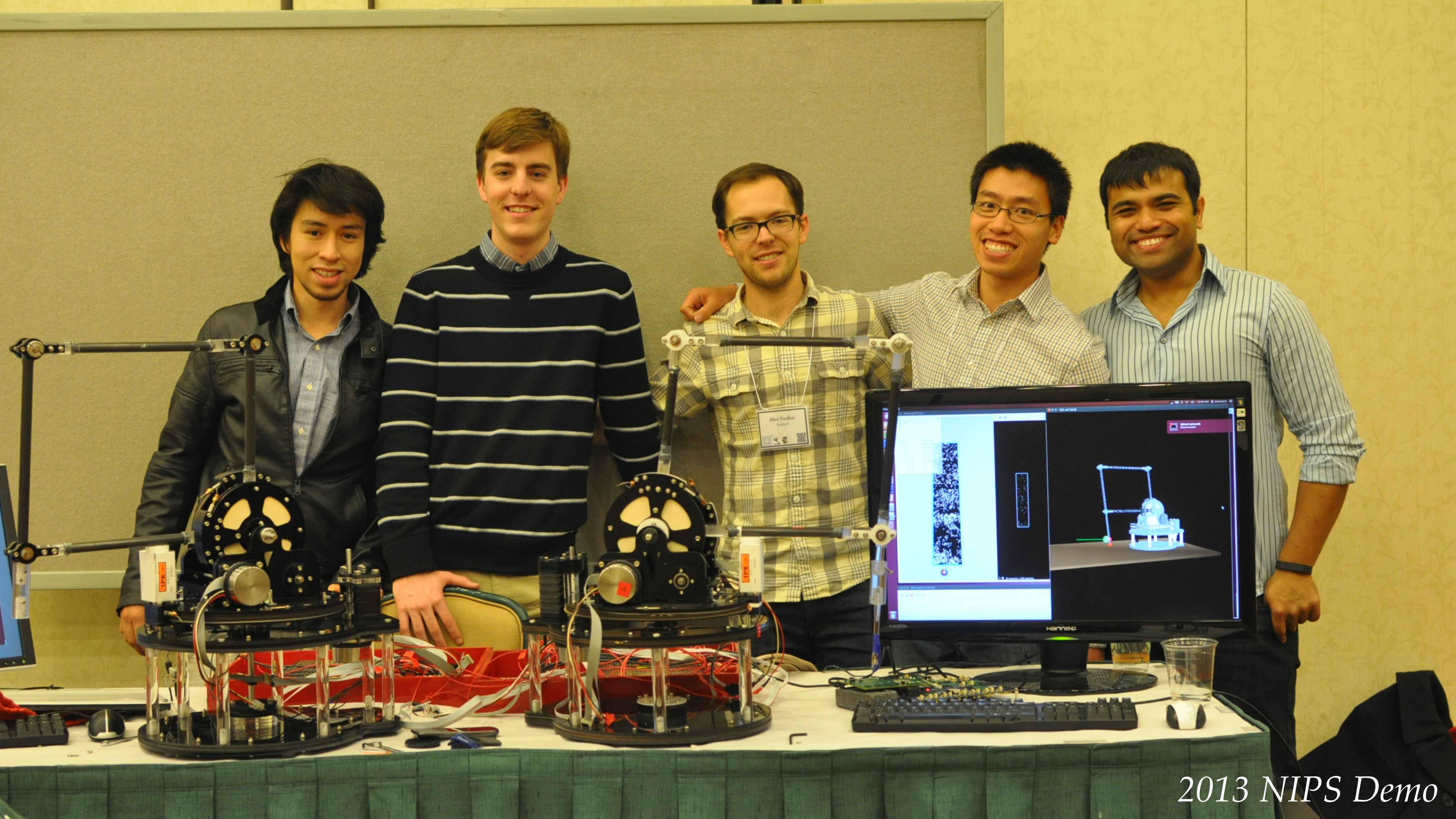




# Digital versus Analog: 1 day versus 1000 yrs



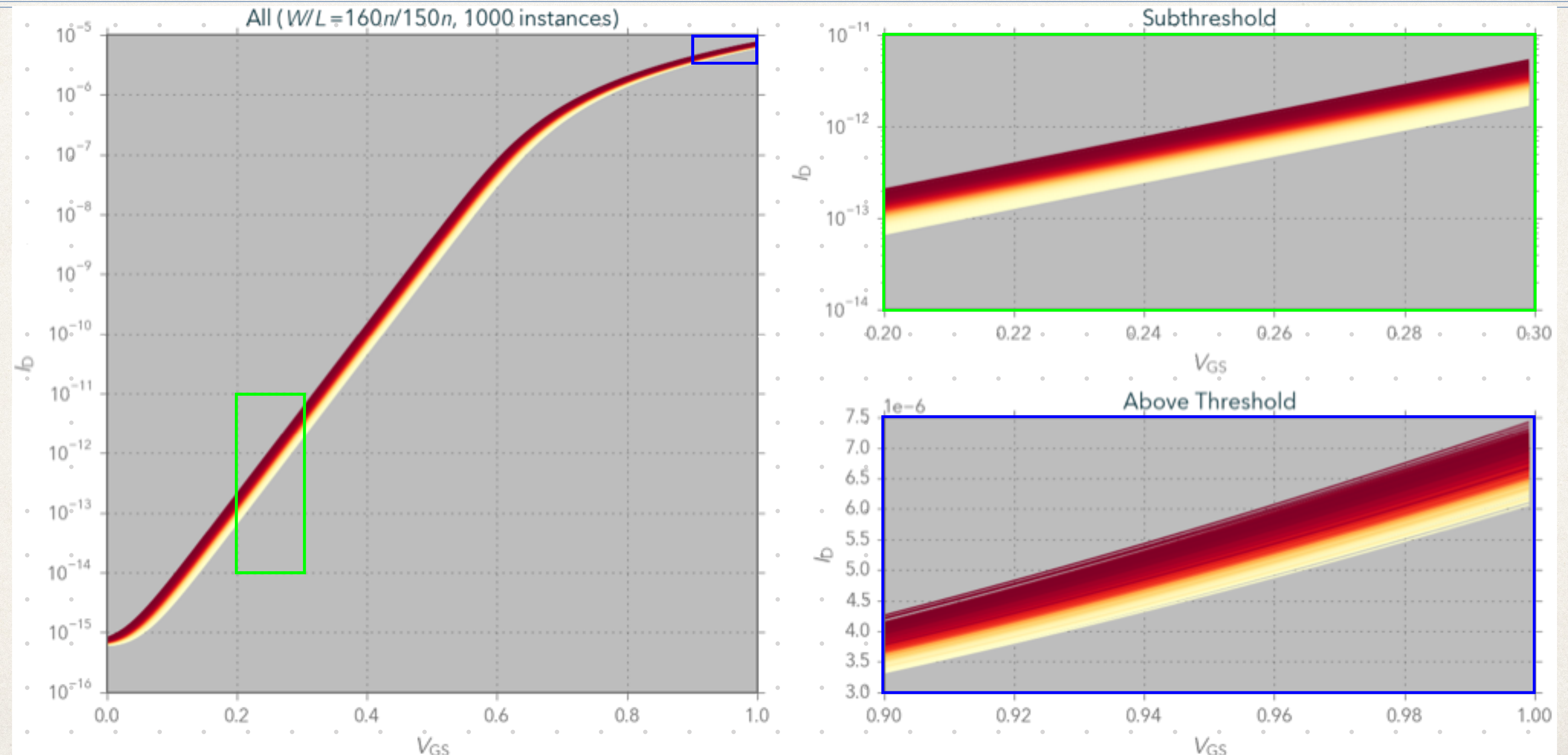




2013 NIPS Demo

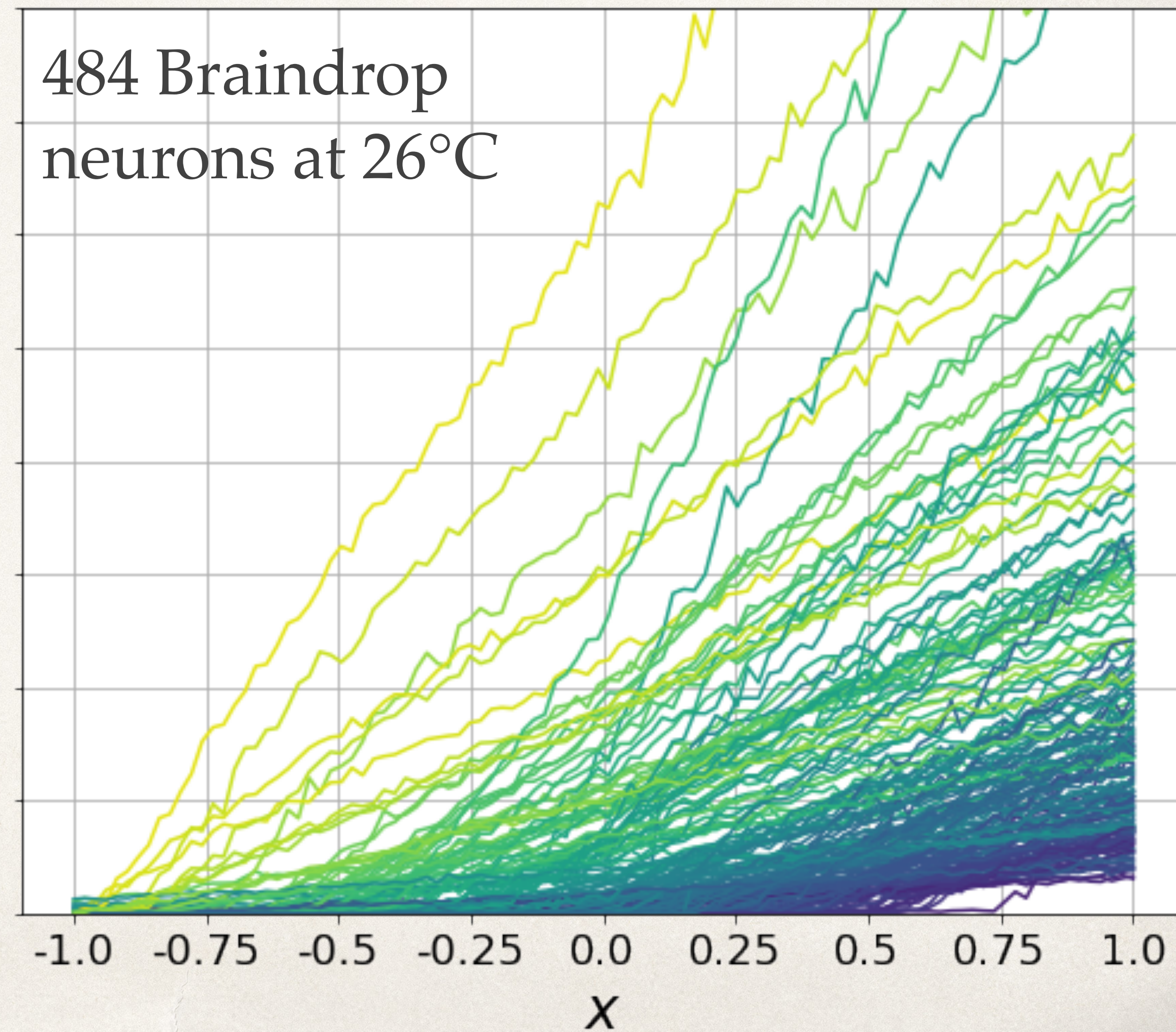
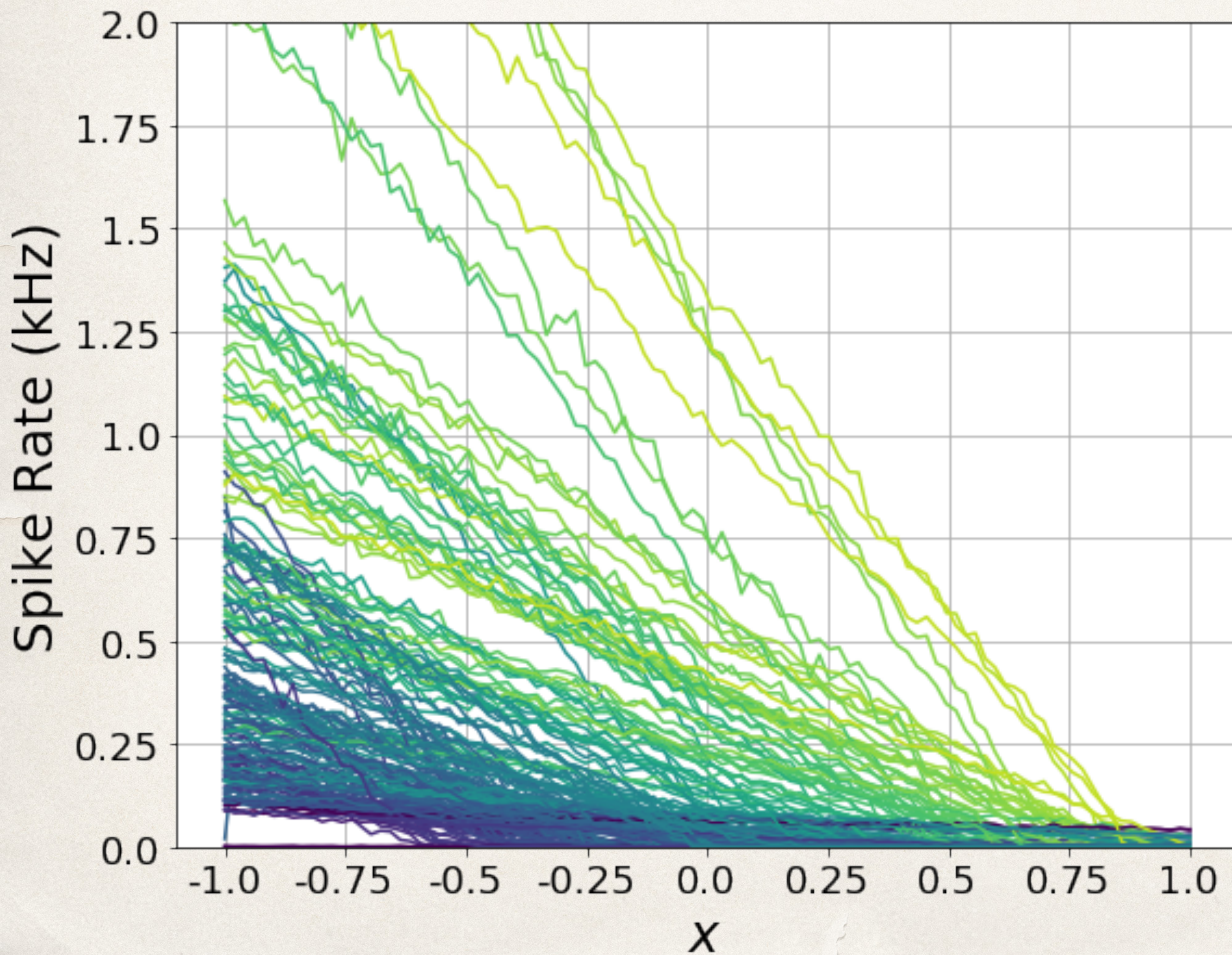


# Analog Challenge I: Heterogeneity



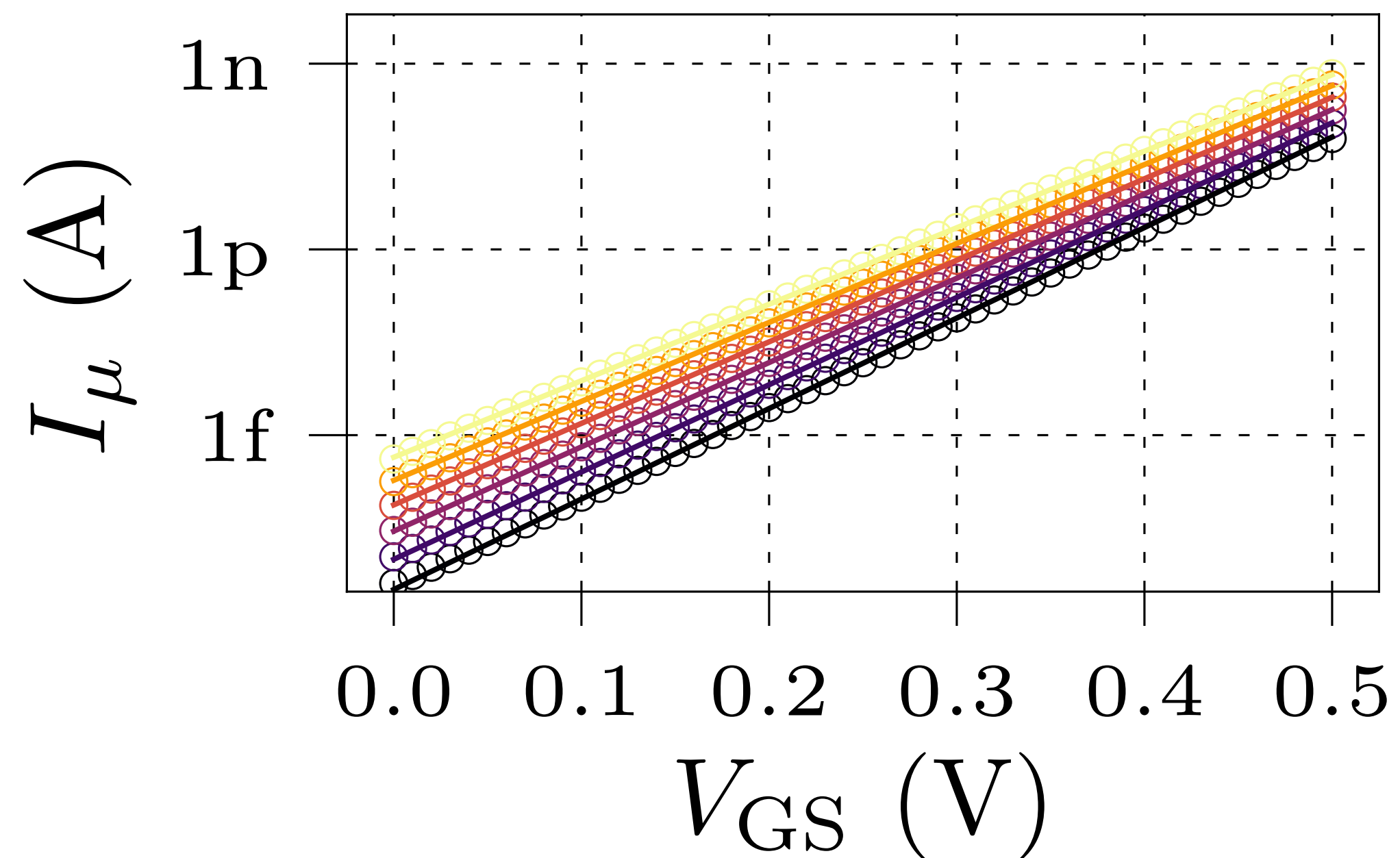


# Silicon neurons' tuning-curves (Braindrop)





# Analog Challenge I: Thermal Sensitivity



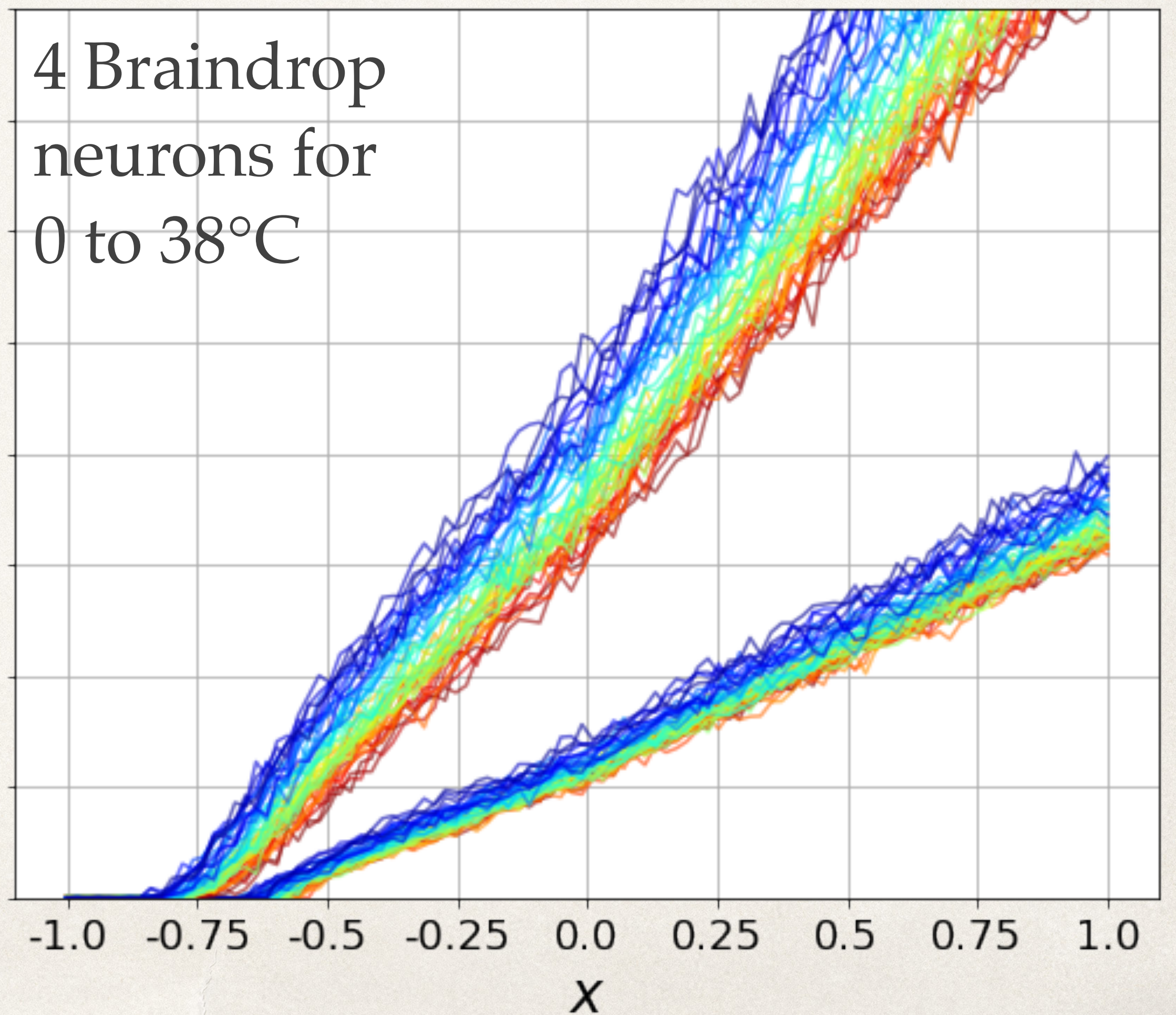
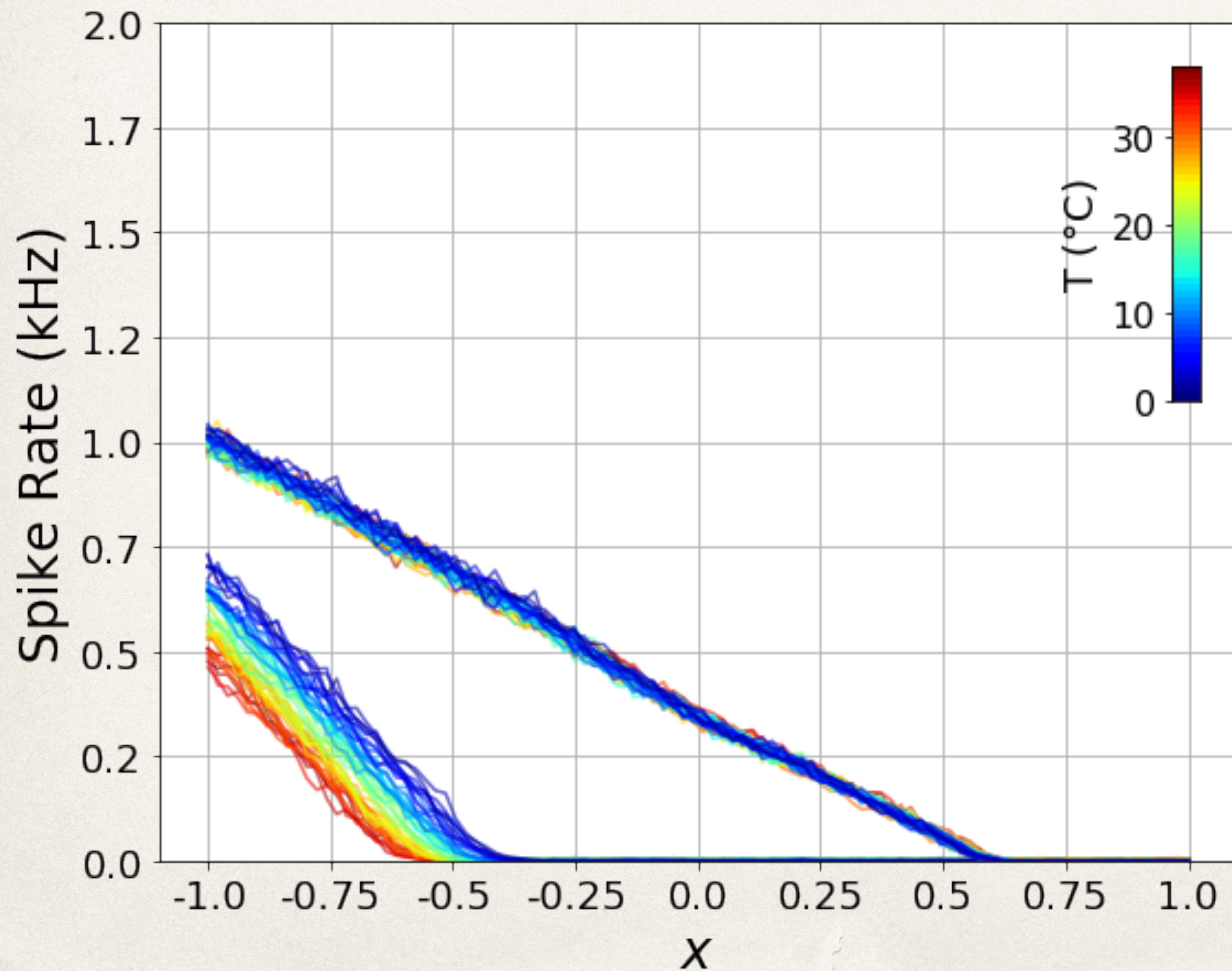
$$I_{\mu}(T) = I_{0_{\text{nom}}} e^{\langle \gamma_1 \rangle \left(1 - \frac{T_{\text{nom}}}{T}\right)} e^{\frac{(1-\kappa)V_{BS}}{U_T}} \times e^{\frac{\kappa V_{GS}}{U_T}} e^{(\lambda_1 \frac{T_{\text{nom}}}{T} + \lambda_2) \Delta V_{DS}} \left(1 - e^{-\frac{V_{DS}}{U_T}}\right)$$

- ✧ A subthreshold transistor's current ( $I_{\mu}$ ) is exponentially sensitive to temperature
- ✧  $T$  is the absolute temperature
- ✧  $U_T = kT/q$  is the thermal voltage
- ✧ Across a 50°C range, the current changes by 1.5 to 3 decades



# Tuning-curves' thermal sensitivity (Braindrop)

*Reid, Montoya, & Boahen 2019*





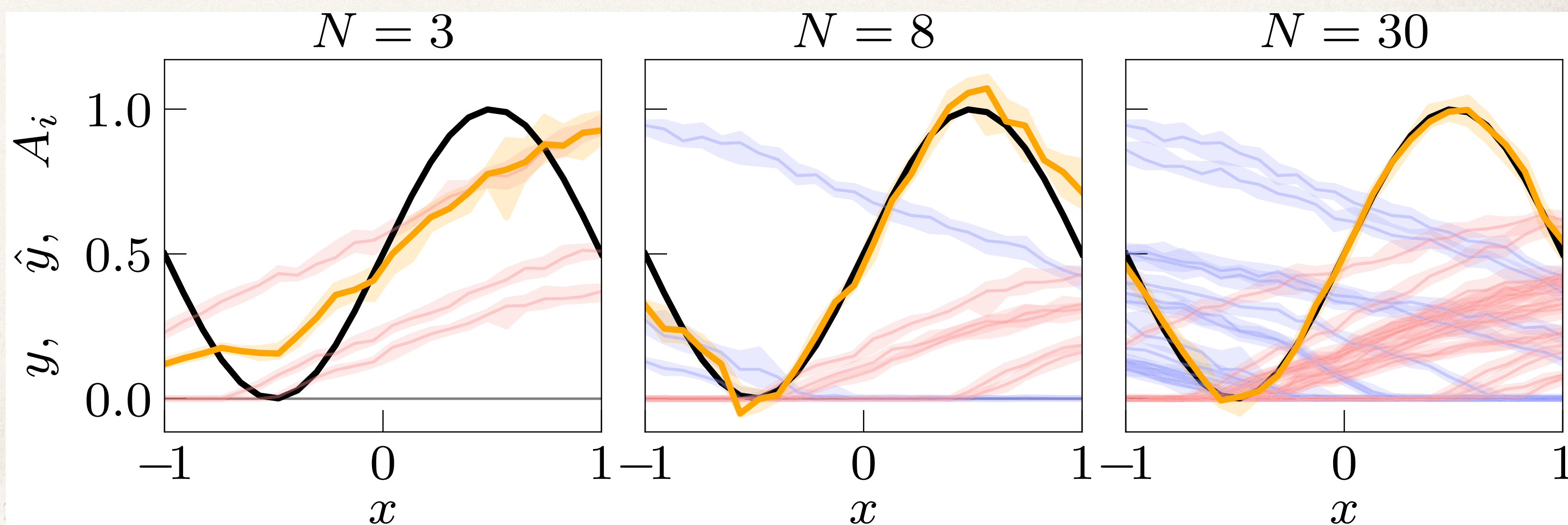
# Approximating functions

- ❖ The desired function  $f(x)$  is expressed as a weighted sum of the neural *tuning curves*  $a_i(x)$
- ❖ The weights—called decoders—are labeled  $d_i$

$$\mathbf{f} = \mathbf{A}\mathbf{d} \Rightarrow \mathbf{d} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{f}$$

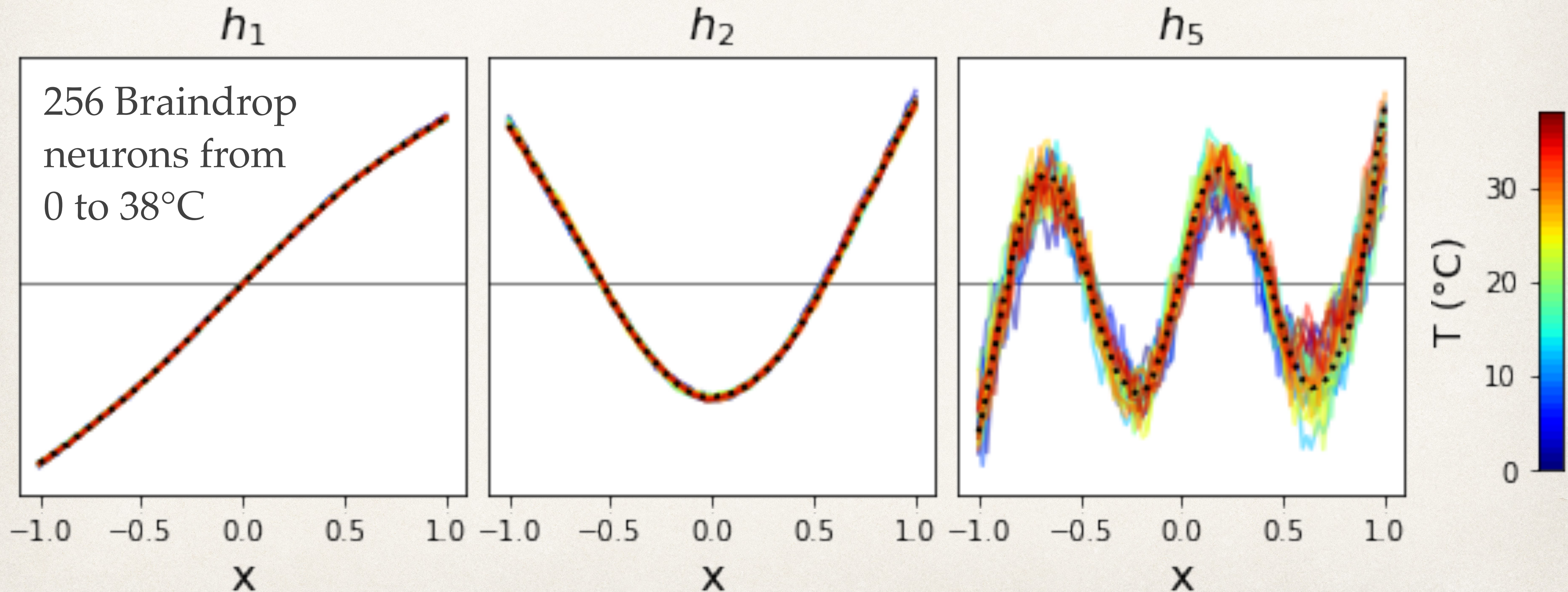
$$\begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_Q) \end{bmatrix} = \begin{bmatrix} \boxed{a_1(x_1)} & \boxed{a_2(x_1)} & \cdots & \boxed{a_N(x_1)} \\ \boxed{a_1(x_2)} & \boxed{a_2(x_2)} & \cdots & \boxed{a_N(x_2)} \\ \vdots & \vdots & \ddots & \vdots \\ \boxed{a_1(x_Q)} & \boxed{a_2(x_Q)} & \cdots & \boxed{a_N(x_Q)} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix}$$

3 to 30  
Braindrop  
neurons at  
26°C





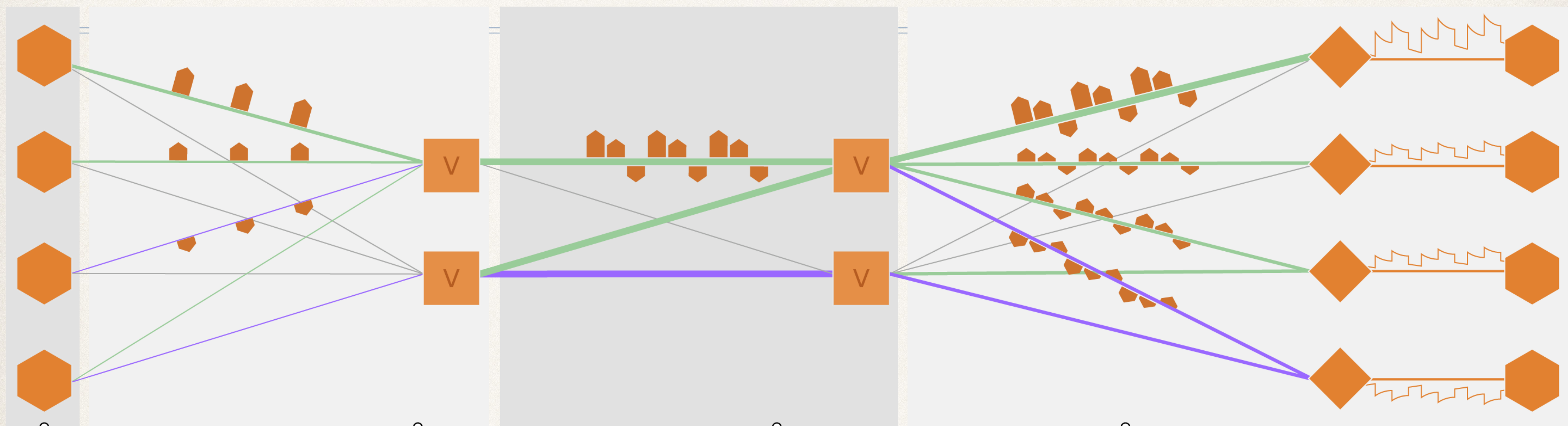
# Thermally robust computation (Braindrop)





# NEF: Decode-Transform-Encode

*Eliasmith & Anderson 2003*



## 1 Spike

$$\langle \delta_{x_i} \rangle_t = [0, \alpha_i(I_i + \beta_i)]$$

Somas emit unit-area deltas  $\delta_{x_i}$  at rates  $\langle \delta_{x_i} \rangle_t$  dictated by their input current  $I_i$ .

## 2 Decode

$$\delta_{y_j} = \sum_i D_{ji} \delta_{x_i}, D \in \mathbb{R}^{D \times N}$$

Deltas are then scaled by their decode weight and merged together.

## 3 Transform

$$\delta_{z_k} = \sum_j T_{kj} \delta_{y_j}, T \in \mathbb{R}^{D \times D}$$

Transform works the same way as Decode.

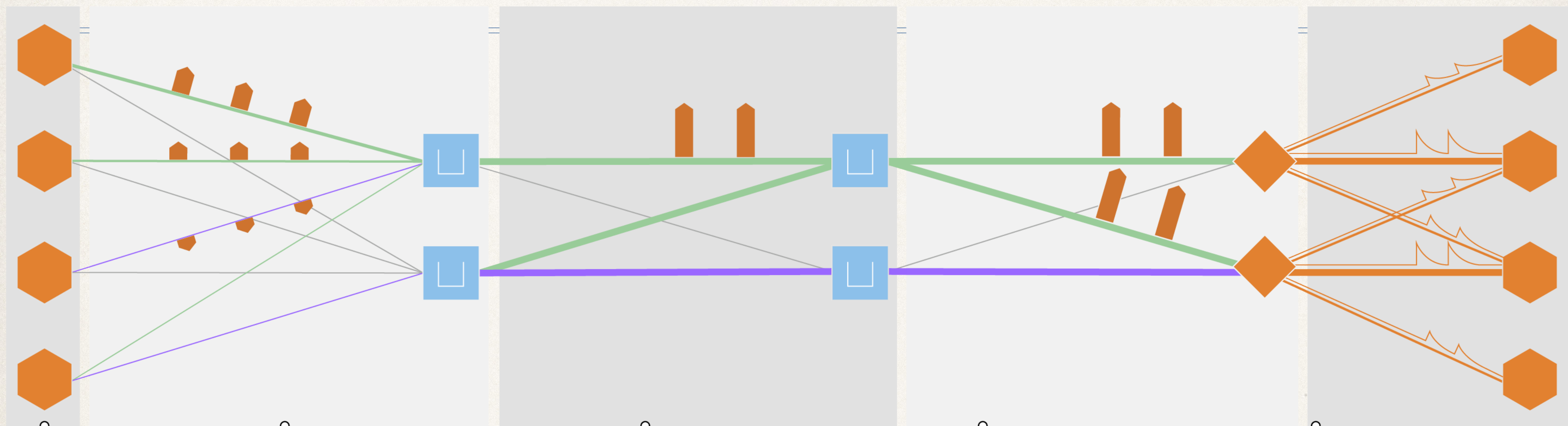
## 4 Encode

$$\tau \dot{I}_l = -I_l + \sum_k E_{lk} \delta_{z_k}, E \in \mathbb{R}^{N \times D}$$

Synaptic filters superpose and low-pass filter weighted deltas to produce output currents ( $I_l$ ) that feed the next soma layer.



# Digital thinning and analog convolving



## 1 Spike

$$\langle \delta_{x_i} \rangle_t = [0, \alpha_i(I_i + \beta_i)]$$

Somas emit delta trains (as in Figure 2).

## 2 Decode

$$\langle \delta_{y_j} \rangle_t = \sum_i D_{ji} \langle \delta_{x_i} \rangle_t$$

$$D \in [-1, 1]^{D \times N}$$

Weighted deltas are accumulated to produce a stream of unit-area deltas.

## 3 Transform

$$\langle \delta_{z_k} \rangle_t = \sum_j T_{kj} \langle \delta_{y_j} \rangle_t$$

$$T \in [-1, 1]^{D \times D}$$

Transform still works the same as Decode ( $T_{kj}=1$  in this example).

## 4 Sparse Encode

$$\tau I_l = -I_l + \sum_k S_{lk} \delta_{z_k}$$

$$S \in \{-1, 0, 1\}^{N \times D}$$

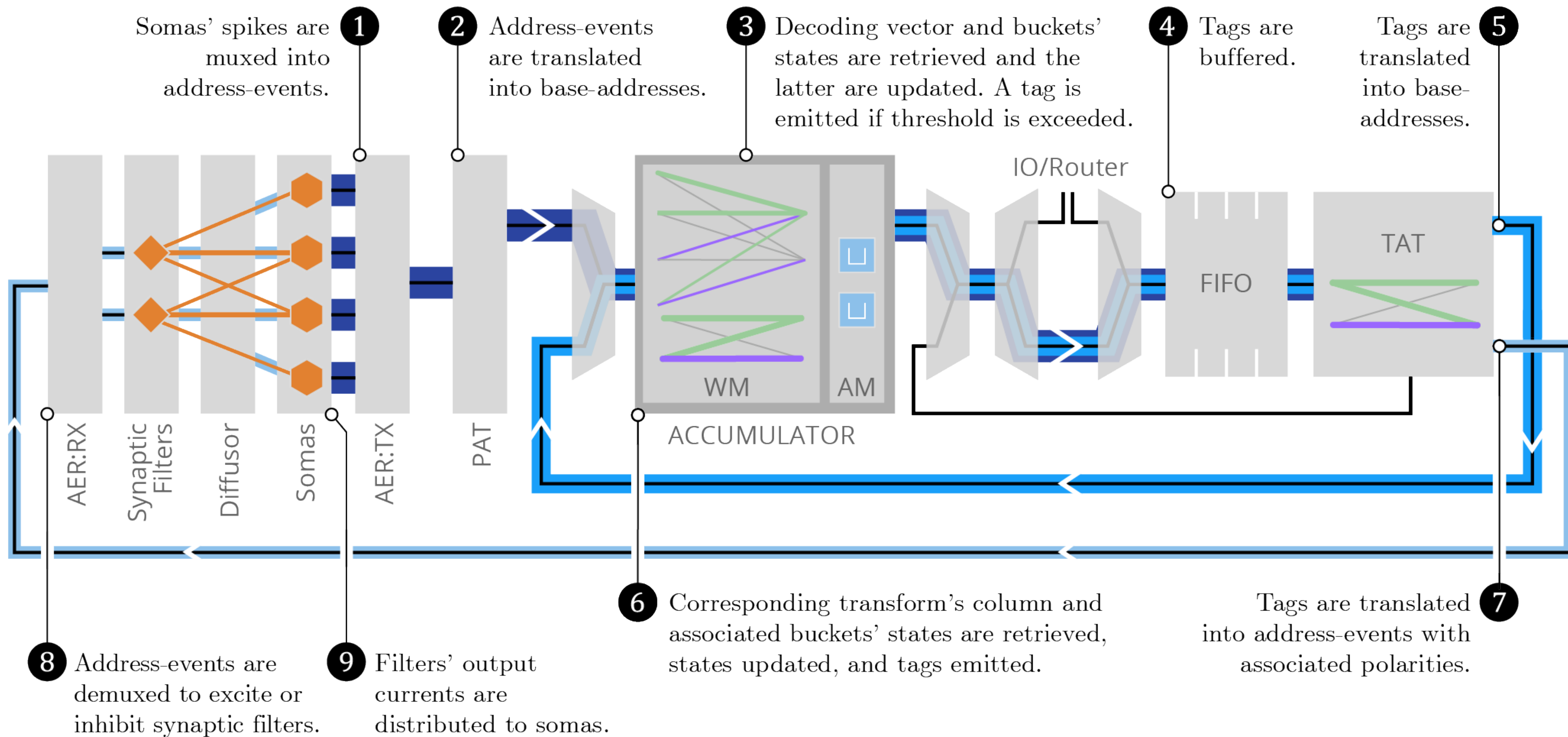
Each accumulator's deltas are sent to a subset of synaptic filters (tap-points).

## 5 Convolve

$$I_m = \sum_l \hat{d}_\gamma(m-l) I_l$$

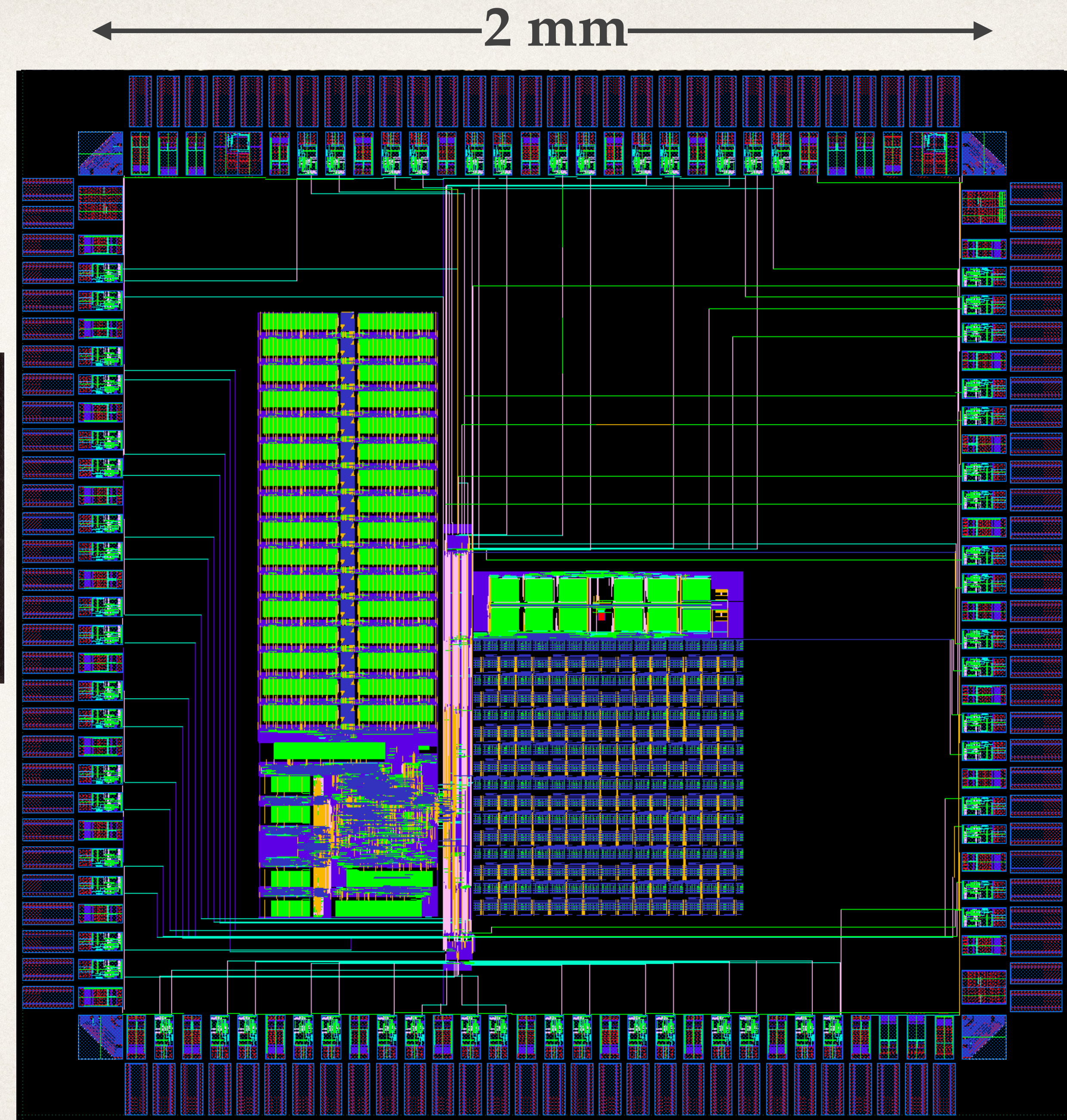
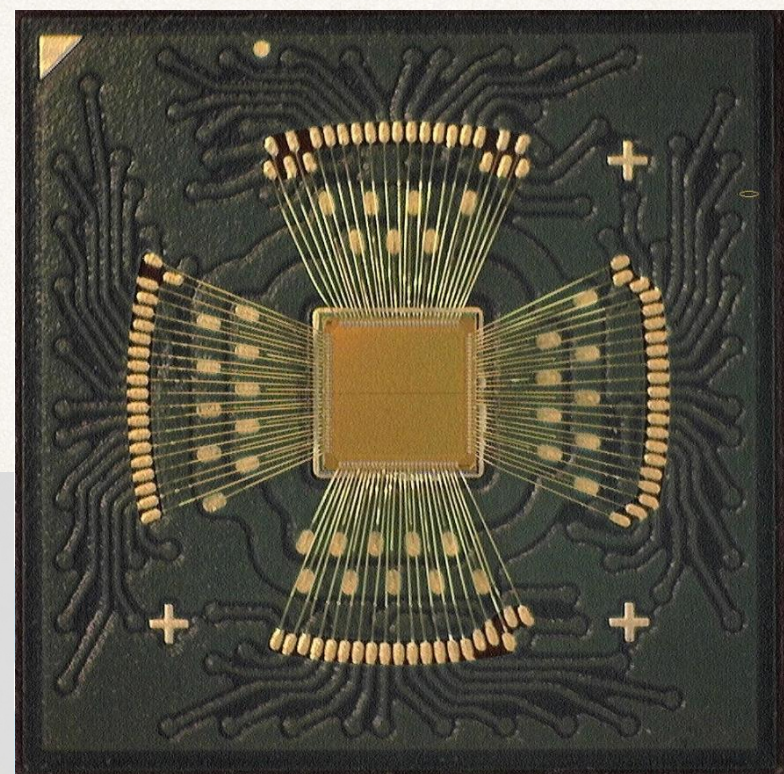
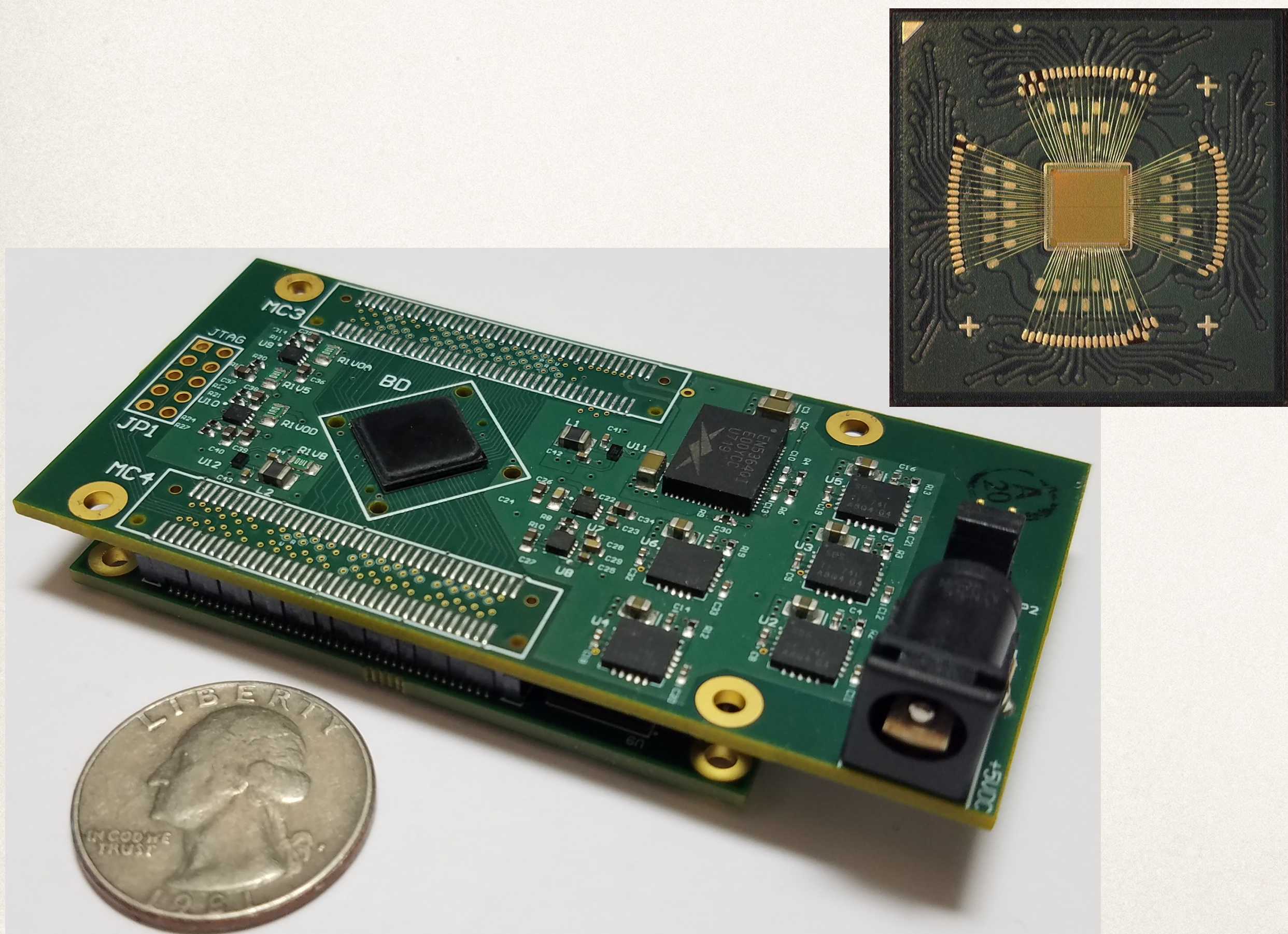
Filter outputs ( $I_l$ ) are convolved ( $I_m$ ) and sent to the next soma layer.







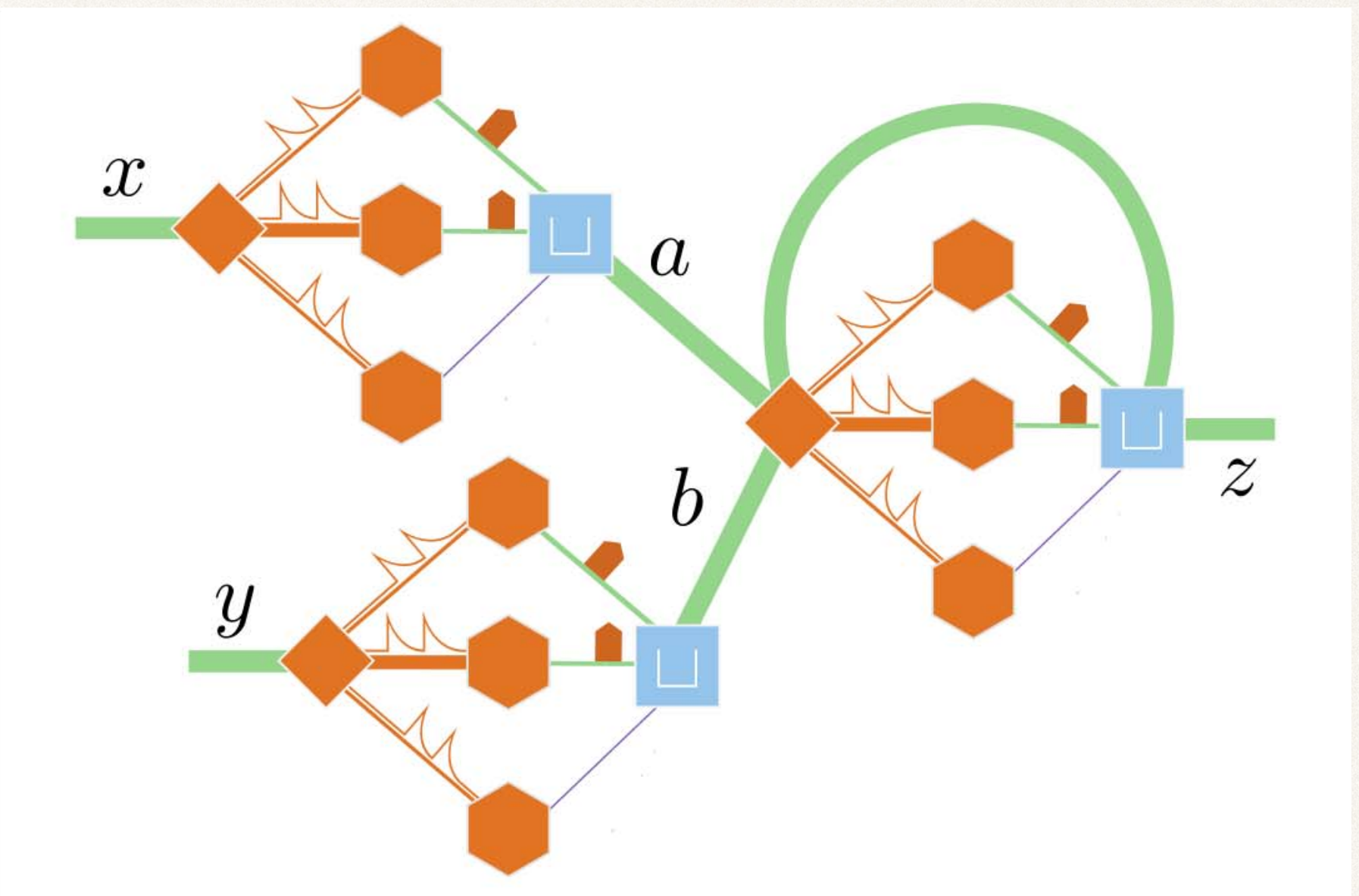
# Braindrop: 4096 neurons in 28nm FDSOI CMOS



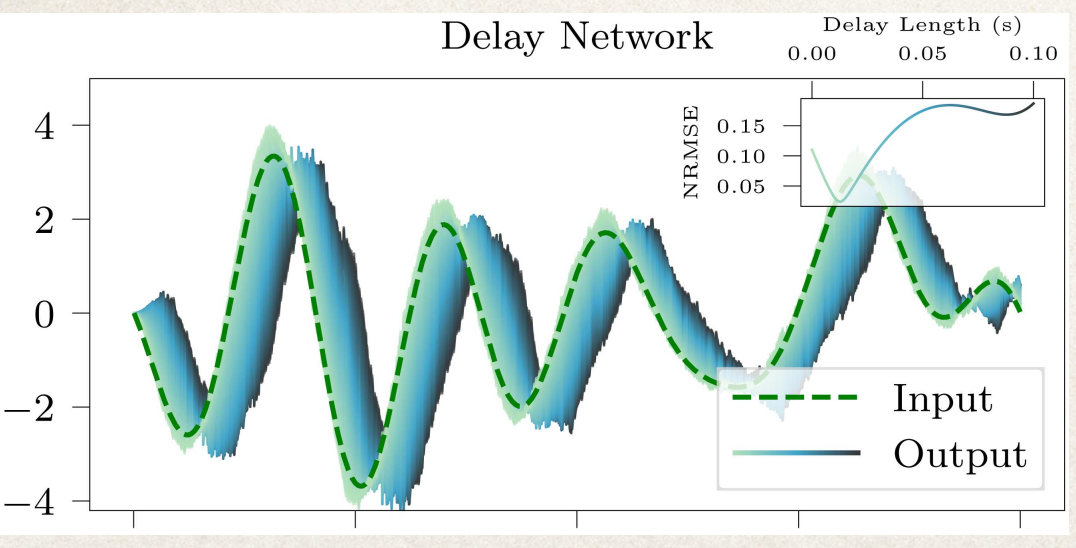
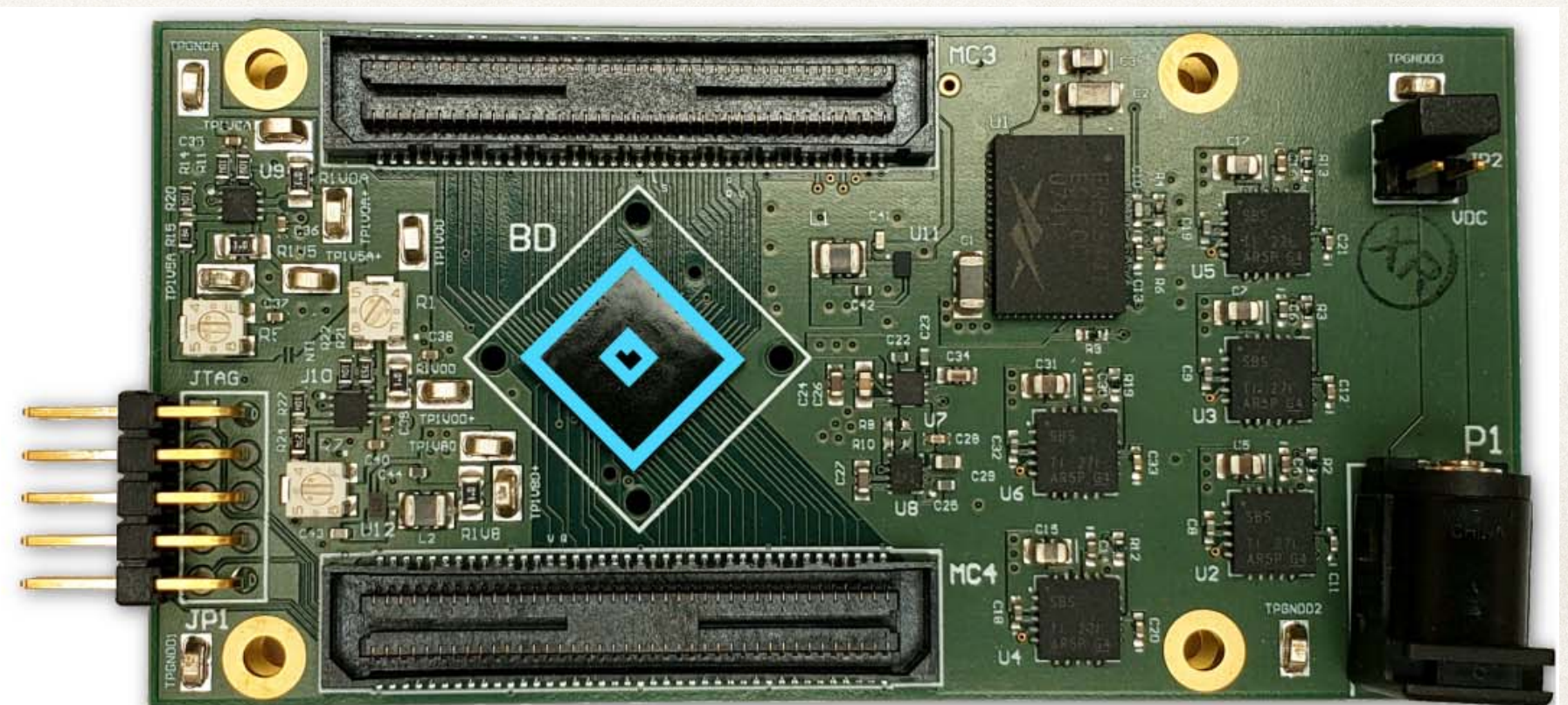
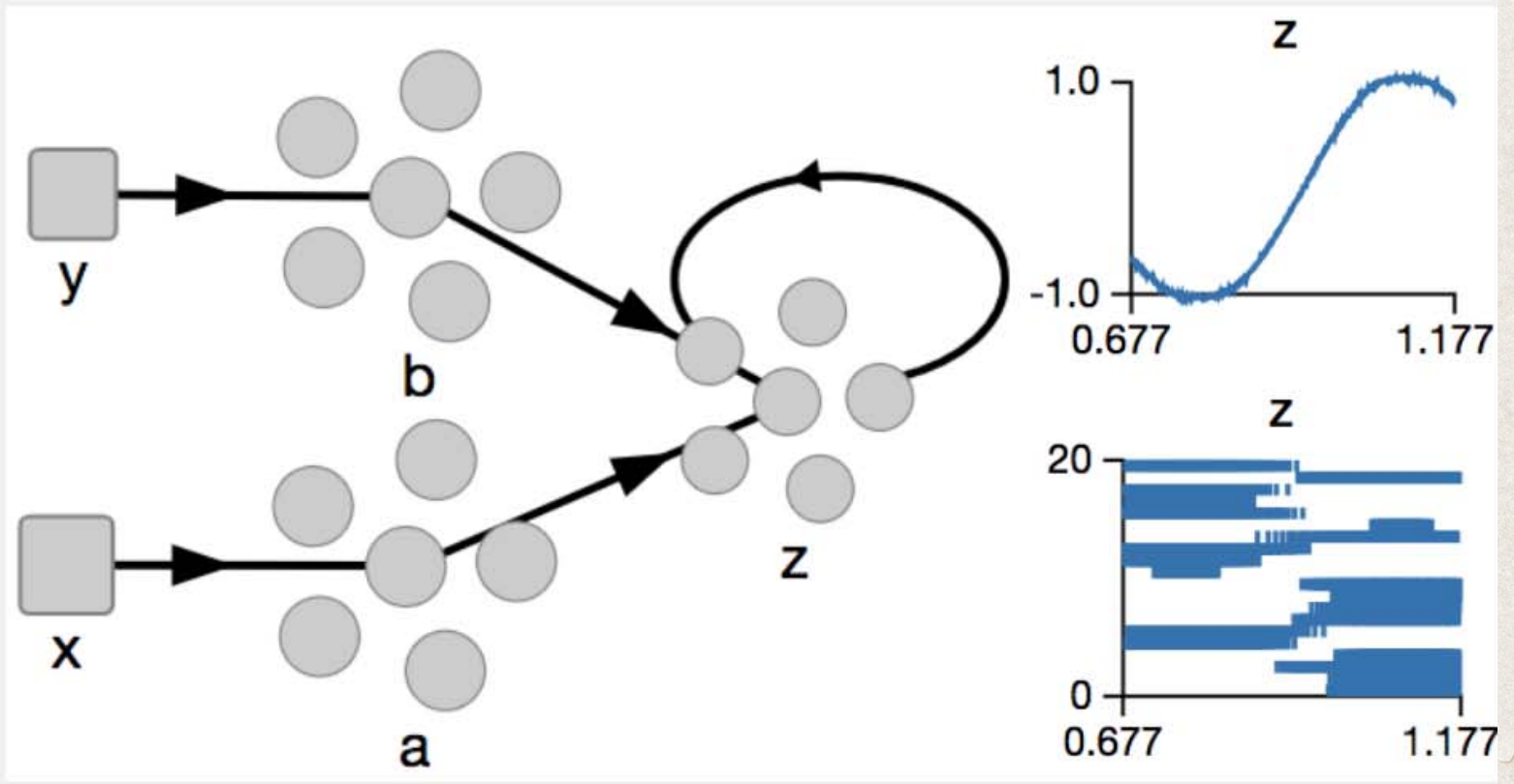


# Programming Environment (Nengo)

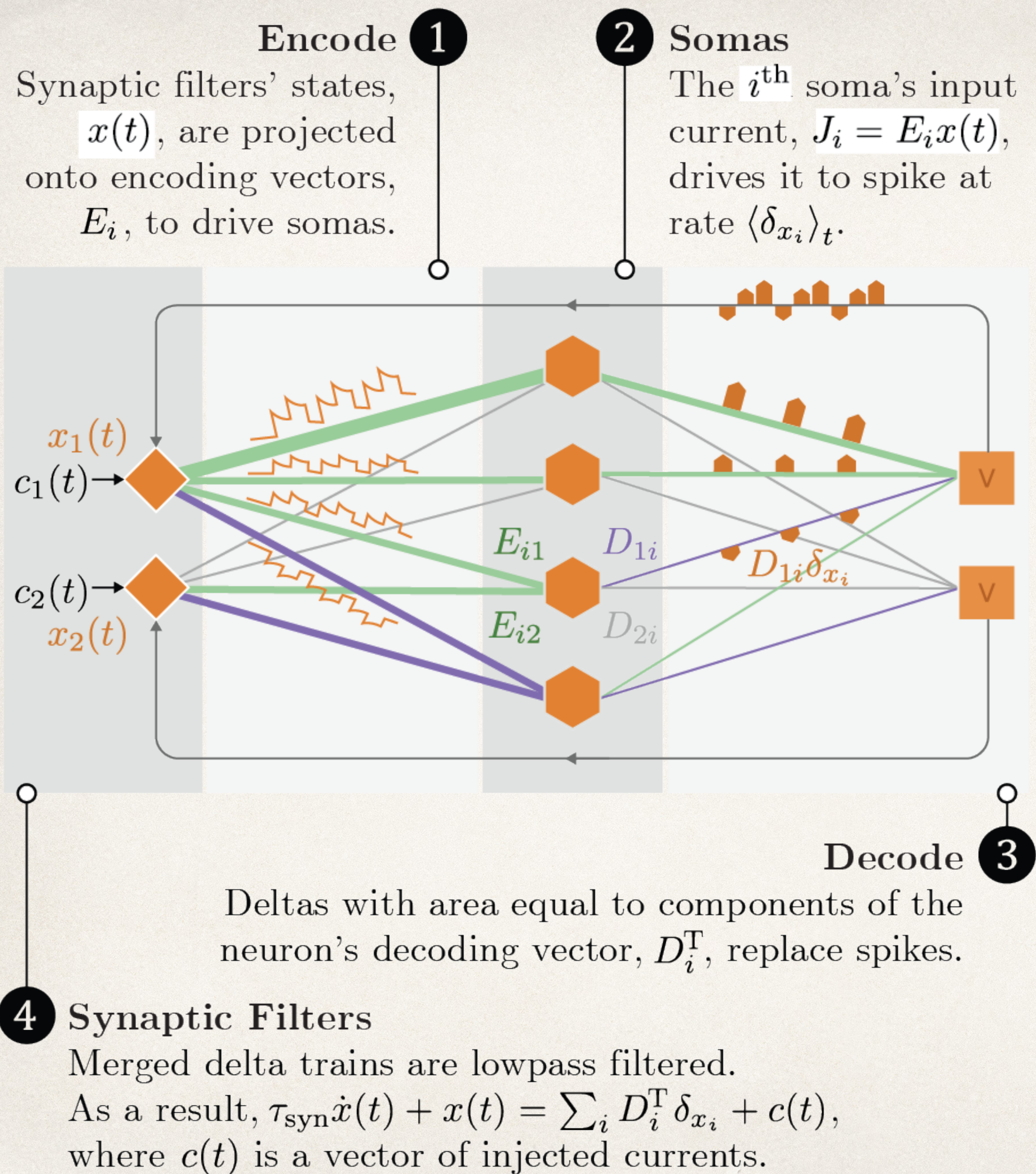
$$\begin{aligned} a &= f(x) \\ b &= g(y) \\ \dot{z} &= h(z) + a + b \end{aligned}$$



```
1 import nengo
2 import numpy as np
3 model = nengo.Network()
4 with model:
5     x = nengo.Node(lambda t: np.cos(2*np.pi*t))
6     y = nengo.Node(lambda t: np.cos(4*np.pi*t))
7     a = nengo.Ensemble(n_neurons=256, dimensions=1)
8     b = nengo.Ensemble(n_neurons=256, dimensions=1)
9     nengo.Connection(x, a)
10    nengo.Connection(y, b)
```







- ❖ To emulate the dynamical system

$$\tau_{\text{dyn}} \dot{x}(t) = f(x) + u(t)$$

- ❖ Choose decoding weights such that, after synaptic filtering,

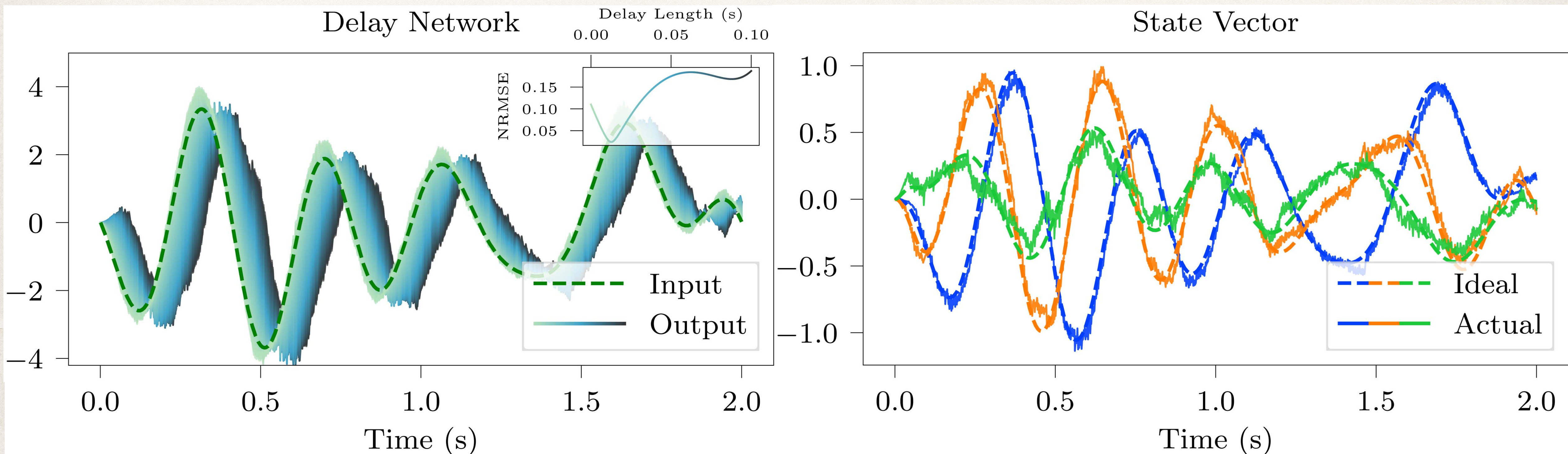
$$\sum_i D_i^T \delta_{x_i} \approx \tau_{\text{syn}} / \tau_{\text{dyn}} f(x) + x$$

- ❖ And set

$$c(t) = \tau_{\text{syn}} / \tau_{\text{dyn}} u(t)$$



# Tapped delay-line (Braindrop)



$$\theta \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Bc(t)$$

$$c(t - \theta') \approx C_{\theta'/\theta} \mathbf{x}(t), \quad 0 \leq \theta' \leq \theta$$

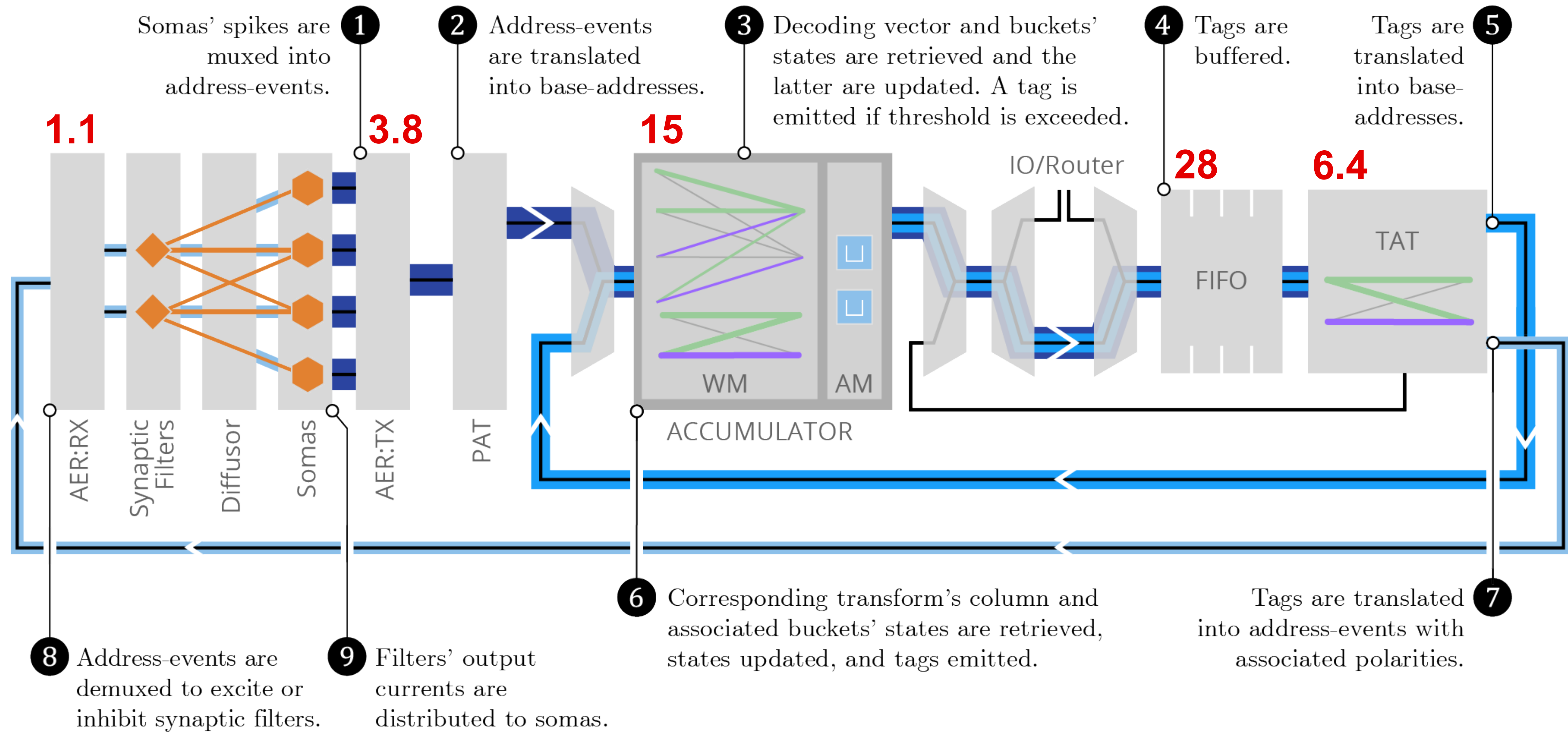
384 Braindrop neurons

*Neckar et al. 2019*

*Voelker & Eliasmith 2017*

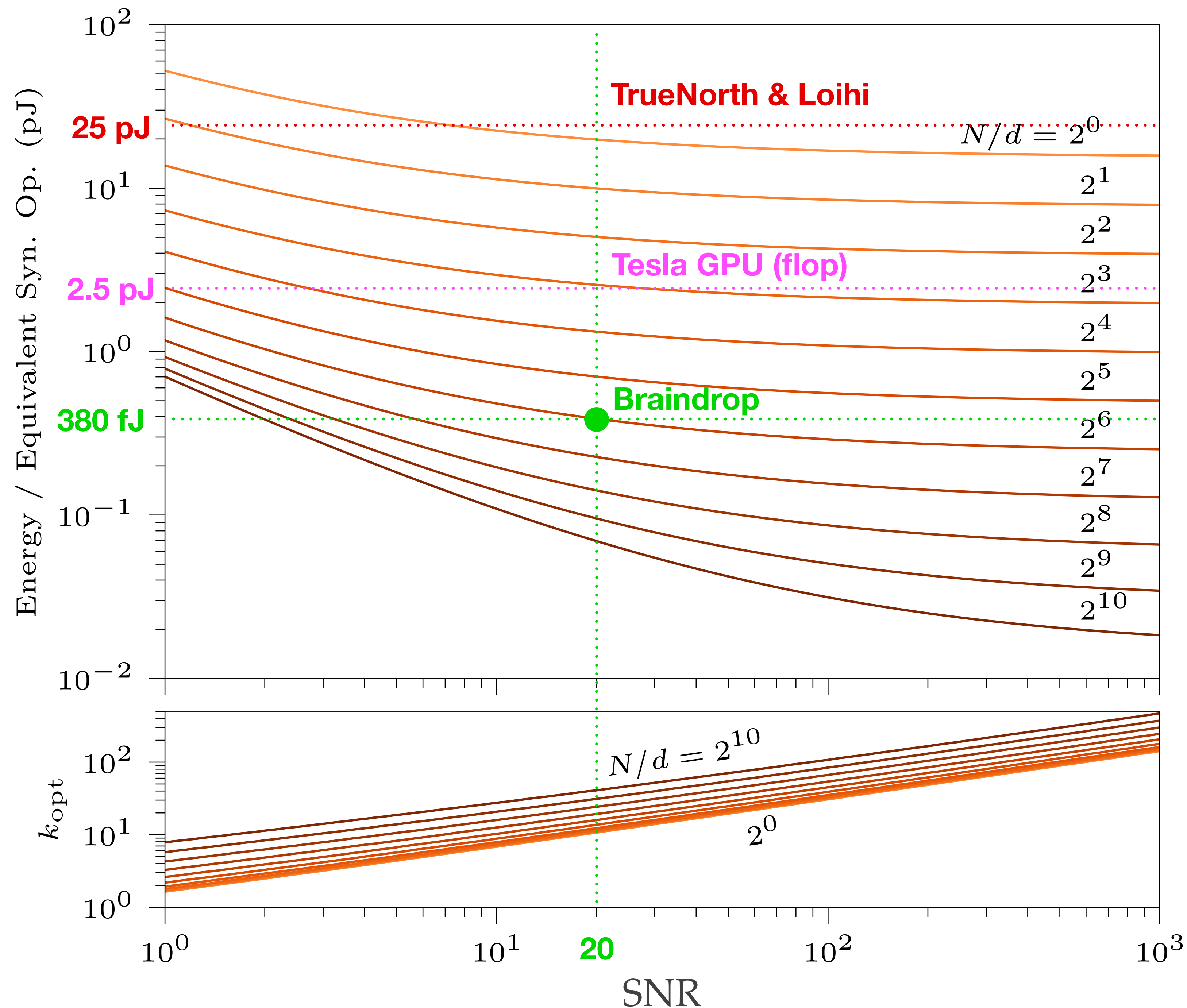


# Measured Energy/op (pJ)





- ❖ Analog convolving fans out  $d$  spike-trains to  $N$  neurons; sparsifies spatially by  $d/N$
- ❖ Digital thinning lets one per SNR spikes through; sparsifies temporally by  $1/\text{SNR}$





# Task performance

❖ Two components:

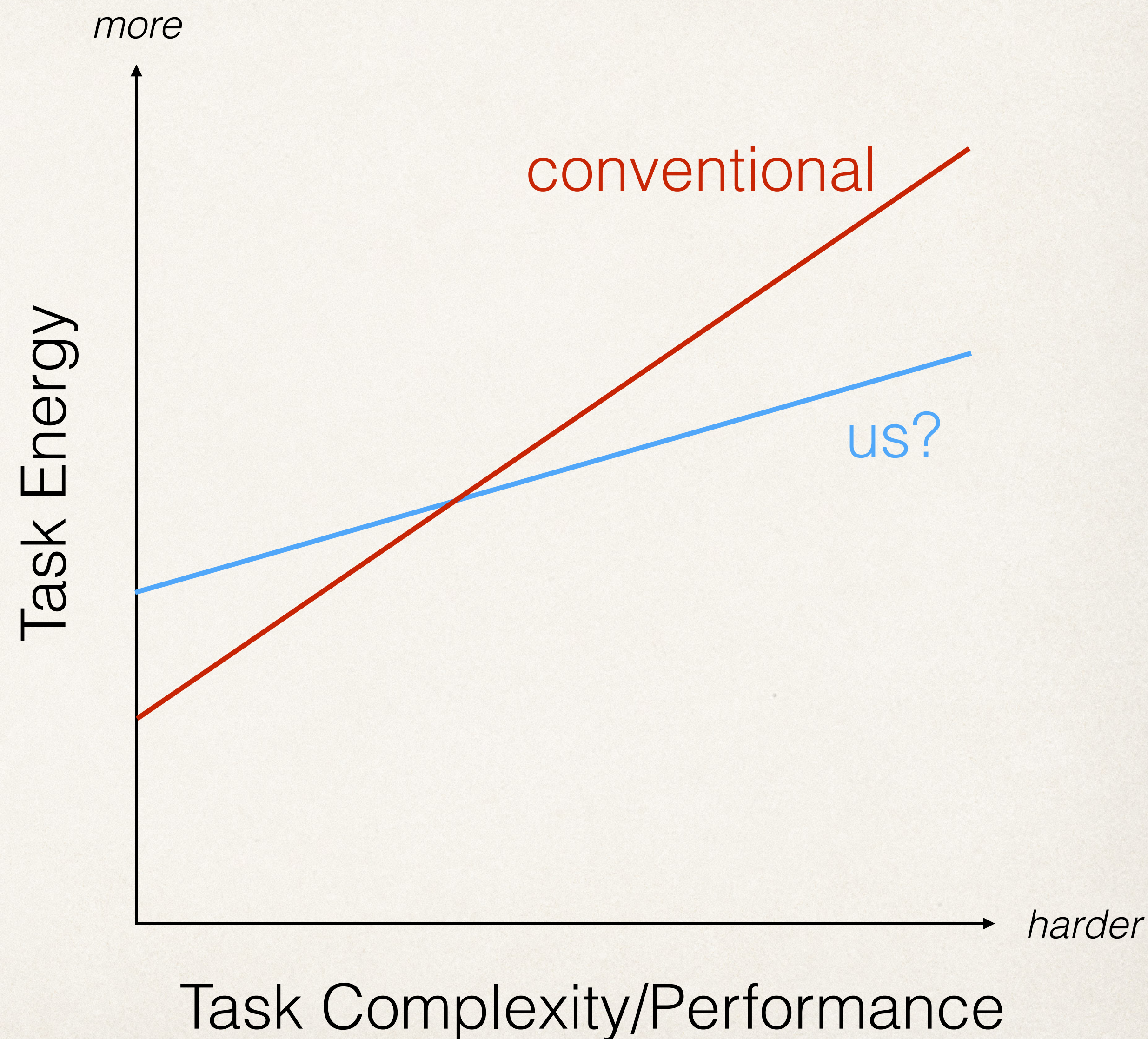
❖ Network design

❖ Hardware design

$$E_{sys}(task) = E_{HW}(R_{net}(task))$$

$$E_{op} = N_{active} N_{conn}$$

$$= (\rho_{active} N)(\rho_{conn} N)$$





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Yale



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## Recent Alumni

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Sam Fok

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Samir Menon

Peiran Gao

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### Waterloo

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Terry Stewart

Aaron Voelker

### Cornell & Yale

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# To learn more ...

J Dethier, P Nuyujukian, C Eliasmith, T Stewart, S A Ellassaad, K V Shenoy, and K Boahen, **A Brain-Machine Interface Operating with a Real-Time Spiking Neural Network Control Algorithm**, *Advances in Neural Information Processing Systems 24*, Curran Associates, Inc., pp 2213-21, 2011.

S Choudhary, S Sloan, S Fok, A Necker, E Trautmann, P Gao, T Stewart, C Eliasmith, and K Boahen, **Silicon Neurons that Compute**, *International Conference on Artificial Neural Networks, LNCS vol VV*, pp 121-128, Springer, Heidelberg, 2012.

S Menon, S Fok, A Neckar, O Khatib, and K Boahen, **Controlling Articulated Robots in Task-Space with Spiking Silicon Neurons**, *IEEE International Conference on Biomedical Robotics and Biomechatronics (BioRob)*, IEEE Press, pp 181-186, 2014.

K Boahen, **A Neuromorph's Prospectus**, *Computing in Science & Engineering*, vol 19, no 2, pp 14-28, IEEE Computer Society, Los Alamitos CA, USA, 2017.

E Kauderer-Abrams, A Gilbert, A Voelker, B Benjamin, and T C Stewart, and K Boahen, **A Population-Level Approach to Temperature Robustness in Neuromorphic Systems**, *IEEE International Symposium on Circuits and Systems (ISCAS)*, Baltimore MD, 2017.

A R Voelker, B V Benjamin, T C Stewart, K Boahen, and C Eliasmith, **Extending the Neural Engineering Framework for Nonideal Silicon Synapses**, *IEEE International Symposium on Circuits and Systems (ISCAS)*, Baltimore MD, 2017.

E Kauderer-Abrams and K Boahen, **Calibrating Silicon-Synapse Dynamics using Time-Encoding and Decoding Machines**, *IEEE International Symposium on Circuits and Systems (ISCAS)*, Baltimore MD, 2017.



*Proceedings of the IEEE, Jan 2019*

## Braindrop: A Mixed-Signal Neuromorphic Architecture With a Dynamical Systems-Based Programming Model

*This paper provides an overview of a current approach for the construction of a programmable computing machine inspired by the human brain.*

By ALEXANDER NECKAR<sup>id</sup>, SAM FOK<sup>id</sup>, BEN V. BENJAMIN, TERRENCE C. STEWART, NICK N. OZA, AARON R. VOELKER<sup>id</sup>, CHRIS ELIASMITH<sup>id</sup>, RAJIT MANOHAR<sup>id</sup>, *Senior Member IEEE*, AND KWABENA BOAHEN, *Fellow IEEE*